THE ANALYSIS AND PROTECTION OF SYSTEMS CONTAINING SERIES COMPENSATED POWER LINES

by

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SUMMARY

Series capacitors are used extensively to reduce the power transfer impedance of long transmission lines. During times of high current, the capacitors are protected from the build-up of high voltages by spark gaps with which they are connected in parallel. This non-linearity affects the behaviour of the system as a whole. Firstly, this thesis is concerned with studying the system behaviour.

A method of analysing the sub-harmonic resonance condition in which line losses are taken into account is proposed, and it is used to find a power system circuit in which operation changes from harmonic to subharmonic mode when a switch is operated. Also described is a method of achieving a transient stability analysis of a two-machine double-circuit series-compensated transmission system with a fault occurring on one circuit. The results are used to compare the effect of two types of protective spark gap. A transient analysis of a single transmission circuit is described, based upon Bergeron's Method of Characteristics. The spark gap type is shown to be instrumental in distorting current and voltage wave-forms. It is generally assumed that spark gap operation shortens the lifetime of the capacitors which it is protecting. This is investigated and is shown to have some theoretical foundation.

Secondly, this thesis is concerned with the problem of finding a suitable distance protection scheme for series compensated lines. A number of possibilities are critically examined by making use of the analysis developed earlier in the thesis. A novel scheme based on definite integrals of

relaying current and voltage was selected and a prototype relay was constructed in the laboratory using mostly analogue, but some digital circuits. Laboratory tests on the prototype relay and simulated tests carried out on the digital computer showed that the scheme meets the speed and accuracy requirements of modern protective relay.

1. INTRODUCTION

The geometry of overhead electrical transmission lines is such that the line inductance is the dominant factor affecting the power transfer capability of a transmission circuit. Power system designs generally require that the transmission line end voltages are kept within 10% of rated voltage and, in order that system stability can be maintained, the phase displacement between these voltages is kept below 30 degrees. For transmission lines of over about 100 miles long, the total series inductance is great and it may not therefore, be possible to operate at economic power levels unless some compensation is introduced.

1.1 Shunt and Series Compensation

For transmission line lengths between 100 and 150 miles it is usually possible to transmit the required power whilst operating within the design limits by connecting shunt compensation equipment at the receiving end of the line. As can be seen from Fig. 1.1. the phase displacement and relative magnitude of the line end voltages can be adjusted by changing the power factor of the received power. It is clear from the diagram that the effectiveness of the shunt compensation in maintaining operation of the circuit within the design limits of voltage ($\pm$ 10%) and phase displacement (30^{o}) depends upon the amount of power being transmitted. If the power, P watts in the figure, is increased by 15% then it is not

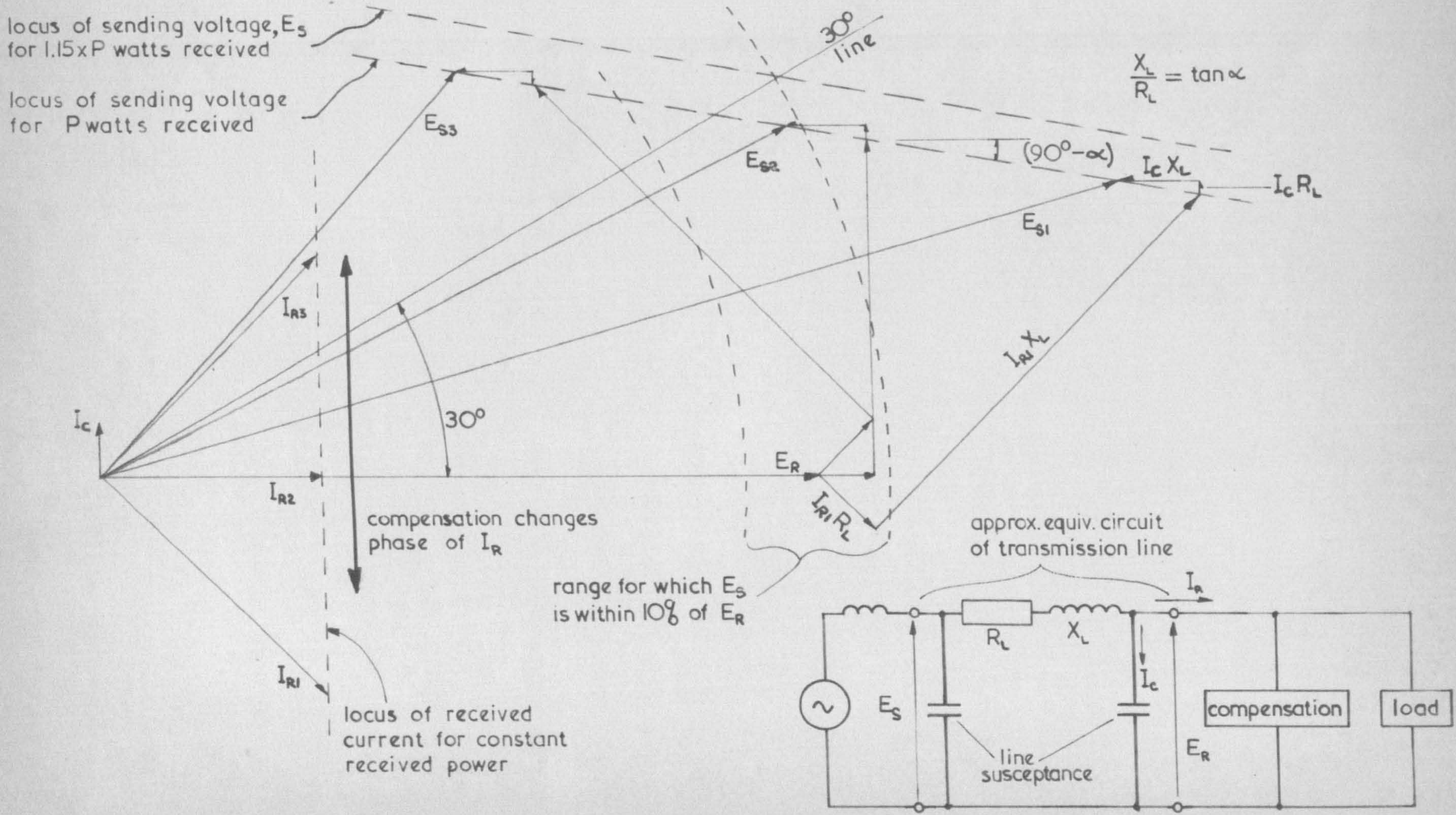


Fig. 1.1 Receiving-end Diagram for Transmission Line Feeding Compensated Load

possible to adjust the shunt compensation in order to maintain operation within the design limits. It is for this reason that shunt compensation cannot solve the problem of transmitting a large amount of power over very long lines.

Series compensation equipment which consists basically of capacitors connected in series with each phase of the transmission line, solves the problem by reducing the power transfer impedance of the line thus reducing volt drop and improving regulation and phase. The way that this is achieved is illustrated in Fig. 1.2. which deals with the same circuit as Fig. 1.1. except that the compensation is series rather than shunt. It is clear that for each of the three load currents considered the phase of the line end voltages is considerably improved by the addition of the series capacitor. The voltage regulation is also improved but some additional shunt compensation, usually shunt inductors, may be required for fine control of the voltage for some low power factor loads. It is significant that with the series compensation illustrated, the received power can be increased by 115% before phase and regulation limits are violated; this compares with a maximum of 15% increase in the same power for the shunt compensated circuit.

Shunt and series compensation are therefore not equivalent. Shunt equipment needs to be automatically controlled and it improves mainly voltage regulation. Series compensation does

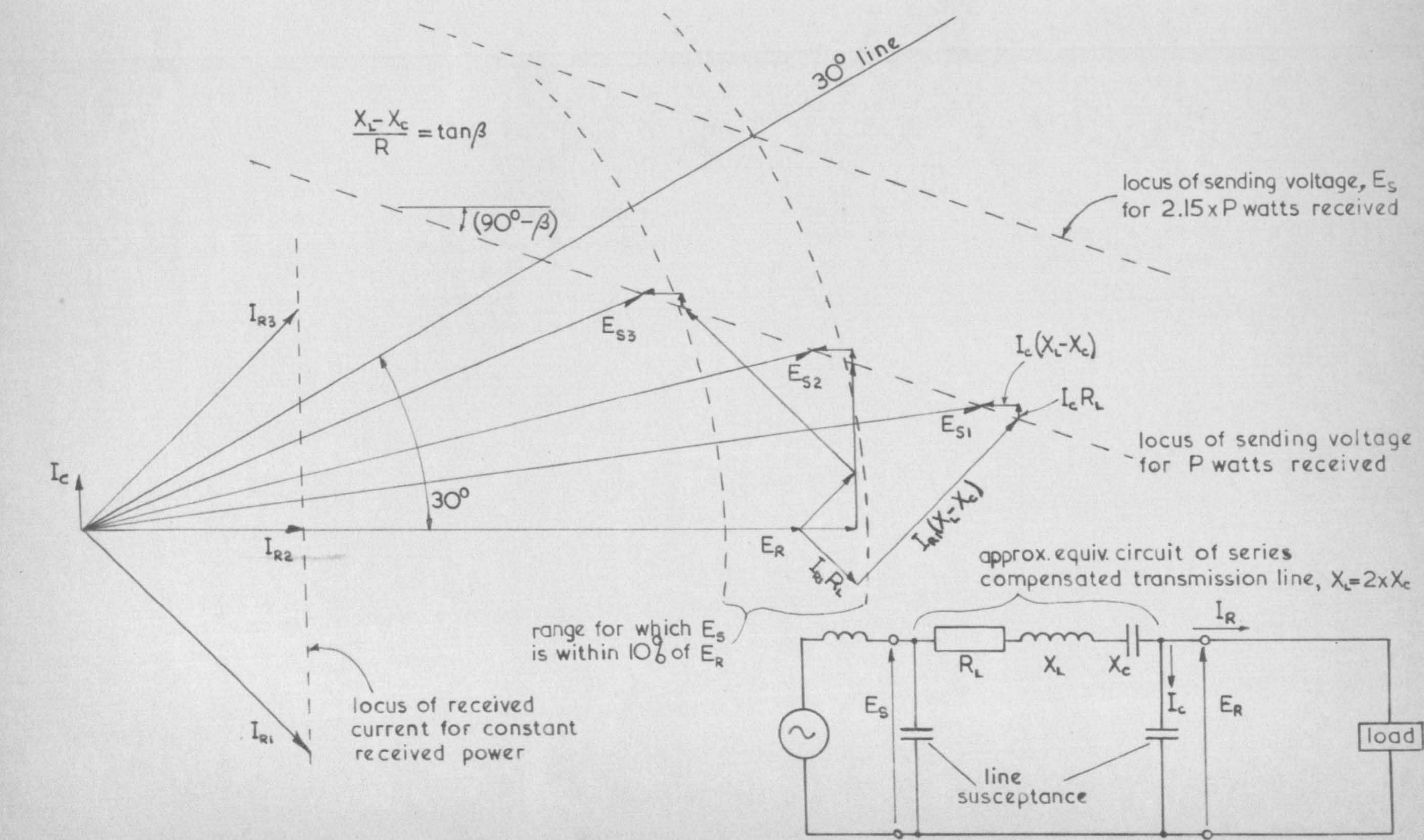


Fig. 1.2 Receiving-end Diagram for Compensated Transmission Line Feeding Load

not require automatic control equipment since it aims to remedy the fundamental, high line inductance, condition. It improves both phase and voltage regulation and can substantially increase the power transfer capability of a circuit. Series compensation is used in countries where power must be transmitted over long distances : America, Canada, Russia and Sweden each have several installations at voltages up to 550 kV and distances up to 400 miles.

Since there are alternative methods available for increasing power transfer e.g. increase in transmission voltage or construction of parallel lines, the decision to install series capacitors must be based upon economic considerations.[1,2] However, certain factors are difficult to take into account in an economic analysis, for instance, the installation of series capacitors can and does form part of planned system expansion. The time required for adding series capacitors is 2 - 2.5 years as against double this time for building a new line. This speaks in favour of the increased use of series capacitors, as the extension of the system can be based on more reliable data, thus lessening economic risk.

The basic series capacitor technology was developed mainly in America[3,4] and Sweden[5] in the twenty years 1930 - 1950. Sweden has relied heavily on series compensation since 1950[6] but America which has a need for strong intersystem tie lines rather than radial feeders has only recently become heavily dependent upon series capacitors.[7,8] Several recently commissioned installations have led to the present situation, which is that most power authorities throughout the world give serious consideration to the use of series capacitors in their major transmission schemes. It has taken nearly fifty years to establish the series capacitor as a powerful, versatile and proven device in the repertoire of the system designer. This long probationary period was due to the many technical problems associated with the equipment and the consequent reluctance of the electrical power authorities to make use of it.

1.2 Technical Problems Associated with Series Compensation[5,8]

The first technical problem to be encountered was the danger of a high voltage building up across the series capacitor due to a high fault current or other disturbance. Since capacitor voltage and current are related by a simple integral formula, a tenfold increase in current involves a tenfold increase in capacitor voltage. It is uneconomic to install series capacitors having a very high voltage rating in order that they can withstand the occasional 'shock' voltage and therefore the capacitors which are usually rated at about 3 per unit voltage, must be safeguarded by a protective short circuiting device during times of unusually high current. Many different designs of short circuiting devices are used,[8,9,10] each type makes use of a spark gap and incorporates some means of extinguishing the arc which develops in the spark gap when the device operates. The arrangement is such that when the peak voltage rating of the capacitor is reached during a fault or other disturbance, the spark gap is triggered and the capacitor is discharged via the spark gap and some external impedance. The non-linearity which is thus introduced, is the cause of some difficulty in any attempt to analyse the system for the period of the disturbance.[11] The analysis of the system, first of all to compare the effect of different types of protective spark gap and secondly to predict the voltage and current waveforms in the system so that the performance of other equipment can be assessed, forms a major part of this thesis.

Another factor of importance is the design of suitable distance type protective equipment for series compensated lines. The problem of measuring fault position when the impedance of the fault loop can be inductive or capacitive and when the capacitor's protective spark gap is likely to operate at the very time when the fault measuring relay should make a decision, has not yet been solved. This matter is also given attention in this thesis and a novel technique for measuring fault position in a series compensated line is critically examined both theoretically and practically.

The occurrence of ferroresonance and also subharmonic resonance in some of the earlier series capacitor installations caused great concern.[12] These phenomena which are due to the non-linear excitation characteristics of iron cored devices such as transformers, proved to be rather difficult to analyse although not difficult to prevent. The ferroresonance condition has recently been fully analysed but no satisfactory treatment of the subharmonic condition has previously been published. This latter problem is therefore examined, producing an analysis[13] and an understanding of a simple circuit which exhibits the phenomena.

Other problems which are introduced by series capacitors concern the electrical machines which are connected to the system. These difficulties[14] are self excitation of induction machines and self excitation and hunting of synchronous machines. These may be overcome by connecting a resistor in parallel with the series capacitor. The value of the required resistor is such that transmission losses are increased by less than 10% and this arrangement also tends to discourage ferroresonance and subharmonic resonance.

1.3 Aims of the Thesis

The first aim of the thesis is to formulate methods of analysing power systems which contain series compensated lines and in particular to consider.

a) the subharmonic resonance condition.[13]

b) the transient stability analysis of a two machine system.[15]

c) the travelling wave analysis of a transmission line.[15]

Secondly the transmission line distance protection problem is discussed[16] and a novel technique for realising satisfactory distance protection equipment is critically examined. It is intended to reach conclusions regarding the most reliable and effective method of applying the technique and thus to contribute to the solution of a major problem associated with series compensated transmission lines.

Since much of the thesis is concerned with analysis it is intended to consider each component of a power system circuit separately before dealing with the system as a whole. Particular attention is given to the capacitor as a circuit element, as a certain amount of information is lacking regarding the behaviour of capacitors in circuits, especially in periods immediately after they are short circuited.

2. CONSIDERATION OF THE ELEMENTS WHICH COMPRISE A POWER SYSTEM

Each of the major elements of a power system in which series compensated lines form a part are considered. The representation of aerial transmission lines for use in the mathematical modelling of power systems is treated first and then consideration is given to the series capacitor installation. Here, concern about the behaviour of the series capacitor when it is shorted by its protective spark gap has prompted the analysis of the capacitor as a circuit element. Lastly, representation of the sources of generation in power system studies is described.

2.1 Transmission Lines

An aerial transmission line consists of three groups of phase conductors, each suspended on insulators from steel towers which are also used to support as many as three earth wires or 'shield' wires. These form part of the neutral current path; they are electrically connected to the towers which are in turn connected to earth mats at the tower bases. Extra phase conductors associated with a second circuit are often carried on the same line of towers. This is the practice in the British 400 kV system.

Due to the electric and magnetic coupling, the behaviour of a circuit formed from any number of the conductors present is affected by the presence of the other conductors. A set of transmission line parameters is required, which can be used to predict the performance of circuits to which the particular transmission line is connected. The widespread use of power line carrier communication and the need for mathematical support for the practice, has led to closer definition of parameters. Realistic studies of travelling waves and switching surges were thus made possible.

A method of allowing for the effect of the earth in communication circuits was produced by Carson[17] and also by Pollaczek in 1926, but the usual configuration of conductors is complicated and only fairly recently have methods of dealing with the algebra been published.[18,19] The remainder of this section is an appreciation of the recent development in line parameter definition. The parameter matrices which are used subsequently in this thesis are included.

2.1.1 Calculation of Line Admittance Matrix, Y

This is a function only of the geometry of the conductors relative to the earth plane because the conductor and earth surfaces may each be regarded as equipotential surfaces. Y has no real part because the air path conductance is negligible. The admittance matrix is defined by :

$I = Y \times V$ where I is a column matrix of conductor currents.

V " " " " " " voltages.

Q " " " " " " charge.

also $I = \frac{dQ}{dt}$ and $V = \frac{1}{2\pi\varepsilon} \cdot B \times Q$

The elements of B, b_{ij}, are calculated according to the conductor geometry. The equipotential earth surface may be replaced by image conductors and therefore :

$b_{ij} = \log_e \frac{(D_{ij})}{(d_{ij})}$ where D_{ij} = distance between ith conductor and the image of the jth conductor.

d_{ij} = distance between ith conductor and jth conductor for $i \neq j$, = radius of ith conductor for $i = j$.

Matrix B has order $3p + q$, where p is the number of circuits and q is the number of earth wires.

Since $Q = 2\pi\varepsilon B^{-1} \times V$, the last q rows of B^{-1} give the earth wire charges and these are not usually required. The last q elements of V are zero since these are the voltages on the earth wires. Consequently, the last q columns and rows of B^{-1} can be discarded. The remaining matrix, B_A^{-1} has order $3p$ and the admittance matrix, Y, is :

$$Y = 2\pi\varepsilon . \frac{dB_A^{-1}}{dt}$$

$$= j2\pi\varepsilon\omega . B_A^{-1} \quad \text{for signals of frequency, } \omega .$$

2.1.2 Calculation of Line Series Impedance Matrix, Z

In general a Z matrix, calculated in the manner which will subsequently be described, will have order 3p + q where again p is the number of circuits and q is the number of earth wires. The equation relating line series volt drop and line current is :

$$I = Z'^{-1} \times V$$

As in the case of the Y matrix, the last q rows and columns of Z'^{-1} can be discarded and the modified matrix of order 3p is re-inverted to give the corrected Z matrix which allows for the effect of earth wires. Matrix Z' is of the form

$$Z' = R_e + R_c + j.(X_g + X_c + X_e)$$

where suffix g refers to the contribution due to physical geometry of the conductors.

suffix c refers to contributions of the conductors themselves.

suffix e refers to the contribution of the earth path.

X_g is calculated using the B matrix used in the admittance calculations i.e.

$$X_g = \frac{\omega\mu}{2\pi}.B, \quad B \text{ is of order } 3p + q.$$

The contribution of resistance and reactance, R_e and X_e, due to the earth return path is calculated using the infinite series developed by Carson. In order to obtain the formulae, Carson solved the wave equation and his solution is a function of frequency, ω. Because of this and also because of the dependence of R_c and X_c on frequency, the term 'Frequency Dependent Parameter' has come into use. In fact the parameters in linear systems do not vary with ω. The variation is more apparent than real. In the case of earth-return impedance, what is really happening is that ultimately there exists a 2-dimensional field, and each of the small elements, in a Maxwell sense, is a linear constant. In Maxwell's equations there is no frequency dependence; the earth-return impedance is really the ultimate limit of a 2-dimensional ladder network of R-L elements. It is the forcing of these inductors and resistors into a single element, $R_e + jX_e$, which causes the frequency dependence. No progress has yet been made in representing earth effects in any way other than as functions of ω. An alternative approach has therefore been followed by researchers recently.[21] The circuit equations have been transformed from time domain to frequency domain, solutions obtained, then inverse transforms performed in order that a time domain solution can be produced.

The formulae for R_e and X_e are :-

$$R_e = \frac{\omega\mu}{\pi} \cdot P \qquad \text{and } X_e = \frac{\omega\mu}{\pi} \cdot Q$$

Each of the elements of P and Q are infinite real series and are given in Appendix 2.1.

In assigning values to R_c and X_c, the components due to the conductors, it is usual to assume that R_c are the d.c. resistances per unit length of the conductors and to calculate X_c in the standard way by the concept of geometric mean radius.[22] This procedure is often adequate for analysis at power frequencies but when higher frequencies are to be considered, the current is more confined to the surface of the outer layer of conductor strands owing to skin effect, and other methods should be adopted. As in the case of the components due to earth effect, both R_c and X_c become dependent on ω. A formula suitable for calculations involving frequencies greater than about 2 k Hz is proved in reference 19.

$$R_c = X_c = \frac{K\sqrt{\rho\omega\mu}}{\sqrt{2}.\ r(n+2)\pi}$$

where K is a constant due to conductor stranding and is about 2.25.

n=number of strands in the outer layer of conductor

r=radius of each outer strand

μ=permiability of strands

ρ =resistivity of strands

Other formulae are contained in reference 22.

2.1.3 Sequence Impedances

Values for sequence impedances are required in many power system calculations. They are the impedances which relate the sequence voltages to the currents. In a transposed line the sequence impedances can be defined. The values used for such tasks as load flow studies, fault level calculations and stability analyses need not be known to a high degree of accuracy and the elements of the impedance matrix, Z, can then be used as follows :-

positive sequence impedance, $Z_1 = Z_e - Z_{m1}$

zero sequence impedance, $Z_o = Z_e - 2Z_{m1}$

zero sequence mutual impedance, $Z_{oo} = 3Z_{m2}$

Z_e is the earth loop impedance and $Z_e = (Z_{11}+Z_{22}+Z_{33})/3$

Z_{m1} is the mutual impedance between lines of the same circuit. It depends upon those off-diagonal elements of Z which relate voltages to currents of the same circuit i.e.

$$Z_{m1} = (Z_{12}+Z_{13}+Z_{23}) /3$$

Z_{m2} is the mutual impedance between lines of different circuits in a double circuit configuration and

$$Z_{m2} = (Z_{14}+Z_{15}+Z_{16}+Z_{24}+Z_{25}+Z_{26}+Z_{34}+Z_{35}+Z_{36}) /9$$

Often the sequence impedances are determined from actual system tests and the elements of matrix Z can then be calculated by re-arrangement of the equations i.e.

$$Z_{m2} = Z_{oo}/3$$

$$Z_{m1} = Z_o - Z_1/3$$

$$Z_e = Z_1 + Z_{m1}$$

2.1.4 Resistance and Inductance Matrices

As already explained, it has not yet been possible to separate resistance and inductance from frequency and if an analytical study requires that these be constant then it is necessary to assume a value for ω. For analytical studies which involve mainly 50 Hz components in the voltages and currents, it can be shown that using this value of frequency for ω does not introduce large errors. When higher frequencies are to be observed as in the study of switching surges, the use of sequence components introduces greater errors.

In section 3.2, the method of analysing the stability problem requires that matrices of R and L and M are known. These are defined as :

R = Real part of Z

L = Diagonal of Imaginary part of $\frac{1}{\omega} . Z$

M = $\frac{1}{\omega}$. (Imaginary part of Z) - L.

2.1.5 Modal Parameters

In the analysis of systems of polyphase transmission lines in which the line susceptances cannot be neglected, long line equations are used. The basic long line equations for a transmission line are of the form:

$$\frac{d^2V_p}{dx^2} = Z \times Y \times V_p \text{ and } \frac{d^2I_p}{dx^2} = (Z \times Y)^t \times I_p \quad \dots \dots \quad 2.1$$

These are transformed into modal equations[18,19,20] in order that the currents and voltages in the 'modal' lines become independent of the currents and voltages in adjacent 'modal' lines. This is achieved by defining modal column matrices V_m and I_m as follows :

$$V_p = S \times V_m \quad ; \quad I_p = Q \times I_m \quad \dots \dots \dots \dots \dots \dots \dots \quad 2.2$$

S and Q must be chosen so that on substitution of equations 2.2 into 2.1 equations are obtained :

$$\frac{d^2V_m}{dx^2} = S^{-1} \times Z \times Y \times S \times V_m \text{ and } \frac{d^2I_m}{dx^2} = Q^{-1} \times (Z \times Y)^t \times Q \times I_m$$

and the matrix coefficients of V_m and I_m are diagonal.

2.1.6 Parameters for Single Phase studies

In the long line analysis described in section 3.3.3. a simplified single phase representation is used. This introduces certain unavoidable errors into the computed transients. The discrepancy is due to the assumption of a single mode of surge propagation having a uniform velocity. In fact, it may be shown[18] that for a double circuit overhead line having 6 phase conductors and one earth wire, 7 modes exist, 6 of which propagate at or near the speed of light with low distortion. The remaining mode, involving earth return, propagates at a lower velocity and is subjected to greater attenuation and distortion dependent on the frequency spectrum of the transient.

For single phase studies the earth loop impedance Z_e defined in section 2.1.3 is used. The line series resistance and inductance are obtained from this by separating the real and imaginary components in the usual way. The line shunt capacitance is obtained from Y, the equivalent element in the line admittance matrix.

2.2 Series Capacitor Equipment

The degree of series compensation and the location of the series capacitor installation are subjects which have received a good deal of attention in the literature. As is the case with most large items of capital expenditure, these questions are usually resolved on economic grounds. The cost of series capacitor installations is currently of the order of £6000/MVAR and a typical installation is rated at 180 MVAR, it occupies 1 acre of ground and has a capacitance of 80μF/phase. The intention in mentioning these figures is to introduce a sense of proportion.

The installations are composed of many separate capacitor units connected in series and parallel. For the purposes of analysing the power system, the installations may be considered as just one capacitor/phase and the capacitors' protective equipment which is also split into sections, can be considered to be present once in each phase also. The series capacitor installation may thus be represented as shown in Fig. 2.1.

The equivalent representation is considered. In section 2.2.1, the capacitors' protective equipment is described and in section 2.2.2 the more detailed aspects of the capacitor itself are given attention.

2.2.1 Capacitor Protective Equipment

Different designs[8,9,10] of capacitor protective equipment are used. Each equipment has the responsibility of protecting the capacitor from being destroyed by the high voltages which

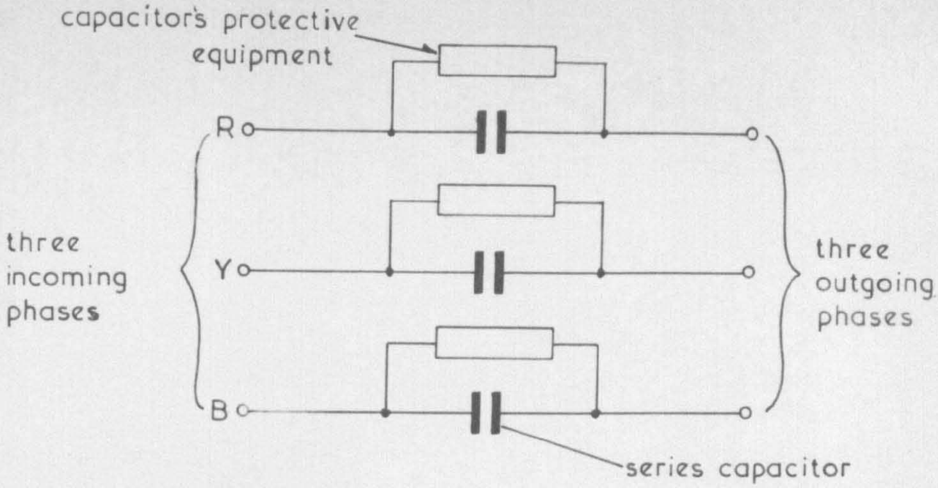


Fig. 2.1 Series Capacitor Installation

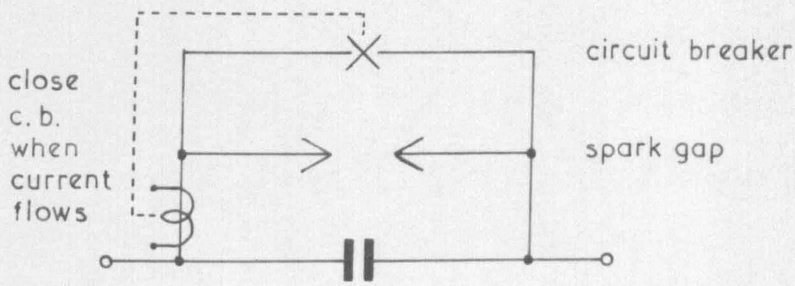


Fig. 2.2 Simple Gap, Simplified Schematic

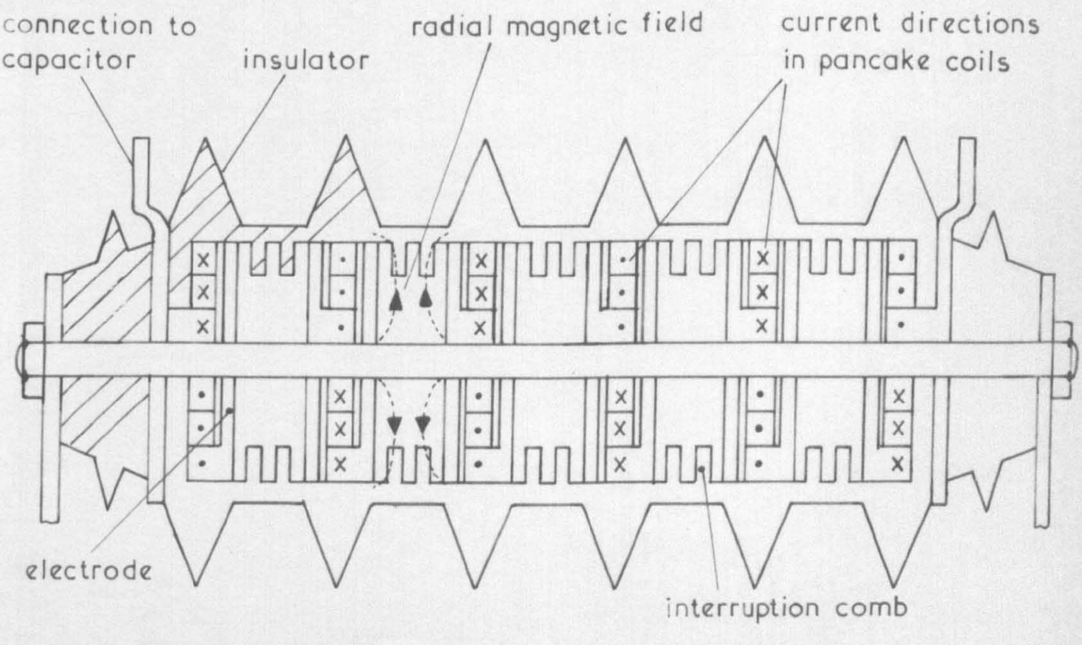


Fig. 2.3 Extinguishing Gap, Magnetic Type (schematic)

would be developed across it if high currents e.g. fault currents, were allowed to flow through it continuously. In principle, a simple two-sphere electrode configuration would, when connected in parallel with the capacitors, provide the required protection, but in practice would be too susceptible to changes in ambient conditions. Also, such a device, when called upon, would need to pass very high currents and consequently it would degrade rapidly.

All capacitor protective schemes on EHV systems make use of the spark gap principle. An auxiliary electrode arrangement is used to sense the critical voltage beyond which the capacitor should not be stressed and this is arranged to trigger a system of main electrodes. The arc which is formed, is the capacitor discharge current summed with the line current, and is caused to 'spin' about some axis by the magnetic field emanating from the conductors leading to the electrodes. This reduces electrode burning by distributing the energy loss from the arc. Once established the arc continues to burn for as long as the arc current is maintained. This is true even through the current is alternating. When there is no longer any possibility of the capacitors being overstressed i.e. when the line current has reduced, the capacitors must be allowed to carry current again. A method of extinguishing the arc is therefore required. The types of capacitor protective equipment can be classified into those which incorporate a simple spark gap and those which have a self extinguishing gap.

2.2.1.1 Simple Gap

The simple gap utilizes an electrode configuration, as described above, together with an auxiliary circuit breaker. These are connected as shown in Fig. 2.2. Current flowing in the spark gap is detected by a current transformer and associated relay and this causes the circuit breaker to close. Due to the relatively low impedance path through the breaker, the spark gap deionizes naturally. The mode of operation of the circuit breaker is different from the operation of those used at line terminations. The breaker adjacent to the spark gap is never called upon to break high currents because these are the very conditions which cause overvoltages on the capacitor. Also, the duty is considerably eased by the presence of the capacitor because transient restriking voltage on the breaker is dependent only on the capacitance value and the line current.

The advantage of installing capacitor protection of this type is that, basically, it costs less than the alternatives. Circuit breakers are often fitted in parallel with the capacitor for other reasons, namely, for the deliberate by - passing of the capacitor to control load sharing between circuits, by-passing when internal capacitor faults are detected and by-passing when subharmonics arise.

The disadvantage of the simple gap is that the operation is fairly slow due to the fact that the plasma, which conducts the current in the spark gap, takes a finite time to deionize and the circuit breaker cannot therefore be reopened immediately. A definite minimum time is therefore associated with this type of equipment and this is typically 2 seconds. The need to reinsert the capacitor after only a short time arises because of the transient stability problem often associated with high currents. The series capacitor can play an important part in maintaining system stability and its early return to the system after having been disconnected by its protection is therefore desirable.

2.2.1.2 Extinguishing Gap

This type of capacitor protection incorporates spark gaps which have the ability to extinguish an arc soon after it has been established. There are two types :

a) Magnetically Quenched Gap[9] This employs the principle used in the original De-Ion circuit breaker. The total gap is subdivided into a number of elementary gaps between flat circular electrodes. These are interleaved with pancake coils having alternating winding directions and connected in series with the electrodes. The arrangement is shown in Fig. 2.3. The magnetic field in the elementary gaps is substantially radial and the axial arc is therefore forced to rotate around the periphery where arc interruption is caused by protruding insulators.

b) Air Blast Gap This does not require the number of gap subdivisions required by the magnetic type. Arc extinction is by a blast of air. Since the operation of this equipment depends upon the opening of an air valve and the flow of air along a pipe, the time to first extinction of the arc is longer than in the case of the magnetic gap. However, for the extinction of any subsequent arcs which may be required due to persistent high current conditions, the air flow is already established and arc extinction is quickly achieved.

The different designs have different merits and drawbacks. The magnetic gap has a simple construction with no moving parts although the mechanical forces between the coils are very great. The air blast gap is more complicated because it requires an air supply at high pressure and also a control system. In some installations the air supply is produced actually on the high voltage platforms where a supply of electricity for driving the compressor motors is obtained from the transmission line itself. A high pressure air supply may be required at a series capacitor station for other functions e.g. shunt connected circuit breakers. Extra compressor equipment may not therefore be required. Since some automatic control equipment is required for the air blast type, gap operation may be made to follow a certain programme. This can eliminate the repeated discharges which take place when an extinguishing gap makes breaking attempts before the proper time.

In comparing extinguishing and simple gaps, the same point about repeated discharges applies. The simple gap allows the capacitor to discharge once only during each disturbance on the power system. The extinguishing gap discharges the capacitor a number of times dependent on the disturbance duration. With the magnetic type at least four discharges are likely. With the air blast type, only one discharge may occur but usually there will be more.

The subject of capacitor discharge is treated in some detail in section 2.2.2 and an attempt is made to explain the alleged reduction in capacitor lifetime[9] resulting from large discharge currents.

The other factor governing the choice of capacitor protection for a particular application is the effect of using the different spark gaps on the transient stability of the system. This matter is given attention in section 3.2 where an analytical method is developed and transient stability studies described for systems, first with simple gaps and then with extinguishing gaps. The results indicate that the extinguishing gap can help a system to remain stable after a fault.

2.2.2. Series Capacitor

2.2.2.1 Degree of Compensation

The degree of compensation is the value by which the positive sequence reactance of a transmission circuit, excluding the load and generation impedances, is reduced by the addition of the series capacitor.

$$\text{compensation, \%} = \frac{X_c \times 100}{\text{Imag }(Z_1) \times l}$$

where l is line length.

In single-phase studies it is the practice to define the degree of compensation based upon earth loop reactance. This has a value somewhat greater than the positive sequence reactance of a transmission line.

The earlier series capacitor installations provided compensation up to 50% but recently schemes with up to 70% compensation have been commissioned. Because series resistances of lines cannot be eliminated and they would begin to dominate the total impedance at high levels of compensation, it is not thought worthwhile at present to operate with more than about 70% compensation. Brief details of some of the major series compensated lines are given in Table 2.1.

2.2.2.2 Series Capacitor Behaviour

One of the arguments used against extinguishing gaps is that they cause the capacitors to discharge repeatedly and it is said that such treatment can shorten the life of the capacitors.[9] A closer look at the behaviour of capacitors in

circuits is

NAME	COUNTRY	LINE VOLTAGE	LINE LENGTH	NO. OF CAPACITOR STATIONS	OVERALL COMPEN-SATION	TOTAL CAPACITANCE MVAR	COMMISSIONED IN YEAR	REFERENCE NO.
Storfinforsen - Hallsberg	Sweden	400 kv	327 mls	1	20%	108	1954	6
Midskog - Goteberg	Sweden	400 kv	284 mls	2	33%	150	1959	6
Kilforsen - Hallsberg	Sweden	400 kv	326 mls	1	50%	298	1963	26
Volga - Moscow	U.S.S.R.	400 kv	370 mls	1	50%	486	1964	10
Cholla - Four Corners	U.S.A.	345 kv	158 mls	1	55%	147	1964	7
Hjälta - Enkoping	Sweden	400 kv	244 mls	2	60%	525	1966	26
NW-SW Pacific Intertie	U.S.A.	550 kv	2,200 (circuit mls)	30	70% (average)	5732	1967	8

TABLE 2.1 BRIEF DETAILS OF SOME OF THE MAJOR SERIES COMPENSATED TRANSMISSION LINES

therefore appropriate. What is required is an equivalent circuit which can be used to assess the behaviour of a capacitor when, for example, it is rapidly discharged. In this way it is possible to determine what proportion of the total energy involved is dissipated within say, the dielectric. Since thermal stresses are thought to be instrumental in dielectric deterioration, which eventually leads to dielectric breakdown, some comments about capacitor life expectancy can be made. Unfortunately, there is a considerable lack of data regarding the mechanisms of capacitor failure and, therefore, firm conclusions relating to life expectancy cannot be drawn.

By way of an introduction, two simple circuits are analysed in section 2.2.2.2.1. Each circuit involves the discharge of a pure capacitor. In the succeeding sections attention is given to the connection and plate resistances and to the characteristics of the capacitor's dielectric. Finally the equivalent circuit which is derived, is considered in a capacitor discharging circuit.

2.2.2.2.1 <u>Behaviour of an Ideal Capacitor</u>

If a resistor is connected to a charged capacitor, the energy contained in the capacitor must eventually be dissipated in the resistor. If the discharge impedance is zero, as it is in the simplified series capacitor circuits used later in this thesis, then the energy in the capacitor must still be dissipated. This is easily shown.

For a capacitor initially charged to V volts and suddenly connected to a resistor R,

$$i = \frac{V}{R} \cdot e^{(-t/RC)}$$

$$\text{energy dissipated in R} = \int_0^\infty i^2 R \, dt$$

$$= \frac{V^2 C}{2} \cdot \left[e^{(-t/RC)} \right]_\infty^0 = \tfrac{1}{2} C V^2$$

This result is not affected by letting $R \rightarrow 0$. Energy dissipation is independent of R.

A second example further illustrates the point. If a capacitor C_1, charged to V_1 volts, is suddenly connected to an uncharged capacitor, C_2 via a resistor, R, then it is readily shown that the voltage V_2 on C_2 is:

$$V_2(t) = V_1(0).(1 - e^{(-t/RC)}). \frac{C}{C_2} \quad \text{where } C = \frac{C_1 C_2}{(C_1 + C_2)}.$$

if E_1 = total energy of system at $t = 0$ and

E_2 = total energy of system at $t \rightarrow \infty$

then $E_1 - E_2 = \frac{1}{2} C_1 V_1^2 . \dfrac{C_2}{(C_1 + C_2)}$

This is the total energy lost from the system. It is independent of the value of R.

In both of the examples, letting $R \rightarrow 0$ results in infinite currents flowing for infinitesimal times. From the point of view of analysing the power system which contains series capacitors, this concept is satisfactory, but in reality the capacitor itself has internal impedance and this dissipates the energy which would otherwise have been dissipated in the zero resistance referred to above. The capacitor's internal impedance is due to resistance in the connections and plates and is also due to dielectric losses. These factors are considered subsequently.

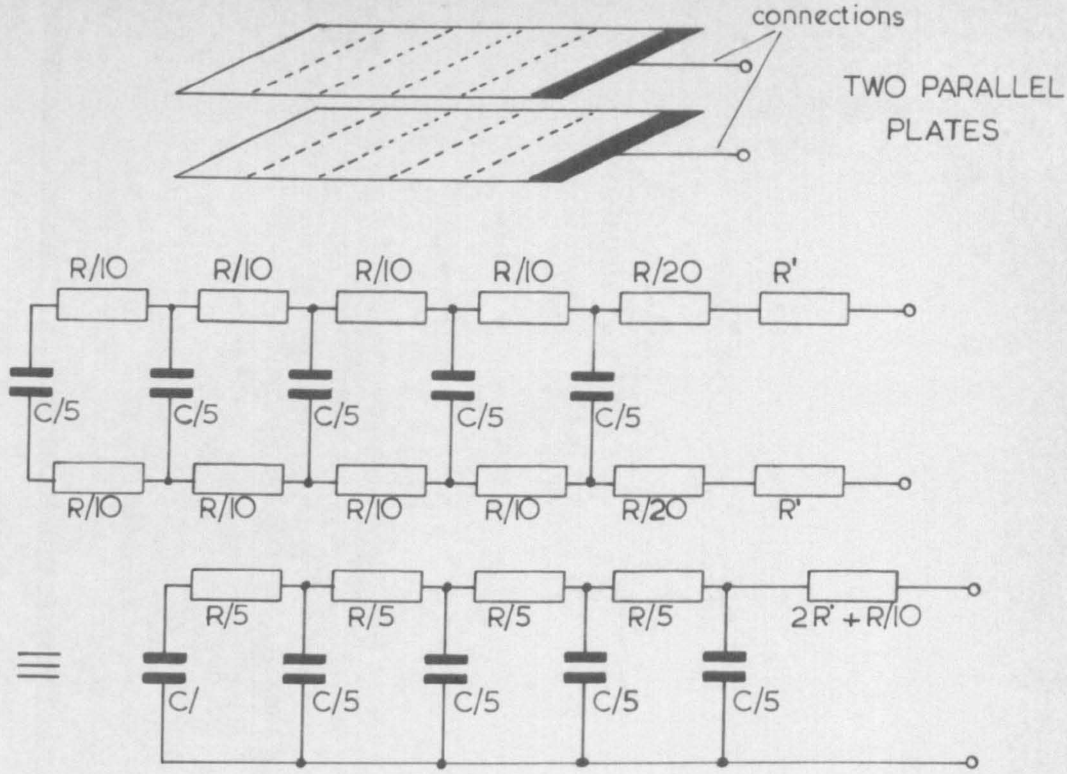
2.2.2.2.2 Capacitor with Resistive Connections and Plates

The approximate equivalent circuit of a capacitor having two parallel plates and separated by an ideal (lossless) dielectric is shown in Fig. 2.4. In practice a capacitor may consist of many pairs of plates in parallel. In such a case the equivalent circuit will be identical to that shown in the figure, but the values of the elemental R's and C's will be scaled by the number of pairs of parallel plates.

2.2.2.2.3 Capacitor with Zero Connection and Plate Resistance but Real Dielectric

Real dielectrics have losses which vary with frequency.[24] The dielectric constant which determines the capacitance of a device also varies with frequency.

In order that wave-forms made up of different frequencies can be dealt with, it is necessary to know how the dielectric will react to each frequency. Many materials may be treated in this way; the earth return medium described in Section 2.1.2 is an example, but in non-linear materials the problem is not so easily dealt with since behaviour may depend on other factors. In materials which exhibit hysteresis, whether magnetic or electric, behaviour depends upon instantaneous position on the hysteresis loop and therefore on the past history of the material. With such materials it is in general impossible to define how a particular frequency will be dealt with. The class of dielectrics which exhibit hysteresis, electrets, are only used in special applications and are never used in series capacitors. The subsequent analysis is not applicable to such materials.

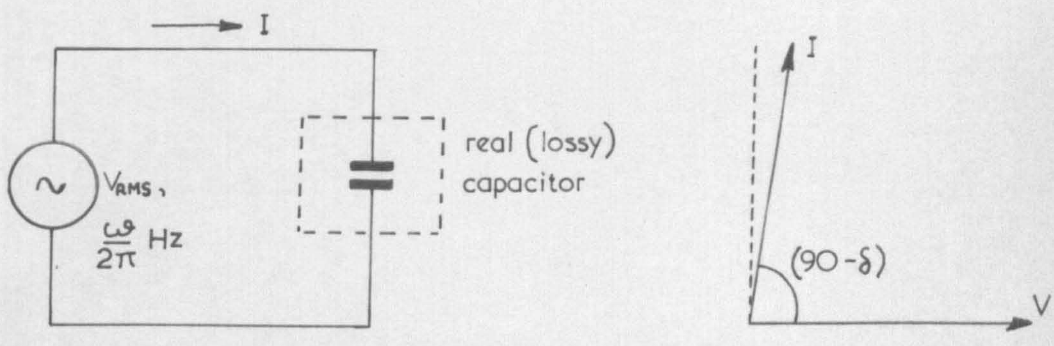


C = total capacitance between plates

R = total resistance of plates

R' = resistance of each connection

Fig. 2.4 Approximate Equivalent Circuit of Parallel Plate Capacitor Having Lossless Dielectric



δ is loss angle

Fig. 2.5 Phasor Diagram for Real Capacitor

All power capacitors use dielectric materials which can be considered to deal with the frequency components of a complex wave as if each frequency appeared separately. This procedure does not introduce errors. An equivalent circuit for the capacitor (with zero connection and plate resistance) is required such that the variation of losses and dielectric constant are identical or fairly closely matched to the characteristics of the dielectric material.

It would be premature to conclude that this equivalent circuit is an ideal capacitor paralleled by a resistor. The frequency response of such a circuit may not agree at all with the observed response of the dielectric because the conductance of the capacitor may not stem from a migration of charge carriers but can represent any other form of energy consuming process. It has therefore been customary[24] to refer to the existence of a loss current in addition to a charging current, non-committally by the introduction of a complex permittivity,

we have $\varepsilon = \varepsilon' - j\varepsilon''$

A parallel plate capacitor having plate area A and plate separation d, has a capacitance, neglecting fringing:

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 A}{d} \cdot \left(\frac{\varepsilon'}{\varepsilon_0} - j\frac{\varepsilon''}{\varepsilon_0} \right)$$

substituting $C_o = \frac{\varepsilon_0 A}{d}$, the capacitance of the same configuration having a vacuum dielectric,

then $C = C_o\,(k' - jk'')$

where $k = \frac{\varepsilon}{\varepsilon_o}$ = complex relative permittivity.

If the capacitor is excited by a voltage, V, at frequency, ω, then a current flows (see Fig. 2.5),

$$I = V.j\omega C = V\omega C_o . (k'' + jk') \quad \text{.. .. ..} \quad 2.3$$

loss angle, δ, is defined by

$$\tan\delta = \frac{k''}{k'}$$

The dielectric conductance, Real($j\omega C$) = $\omega C_o k''$ represents all dissipative effects in the dielectric. These include migration of charge carriers and frequency dependent losses such as the friction accompanying the orientation of dipoles.

The variation of k' and $\tan\delta$ with frequency are important characteristics of a dielectric material and data is available for many materials. The characteristics of Pyranol* are typical. This material is used in modern power capacitors for the impregnation of the paper used for the dielectric. It is sufficient at this stage to consider the general form of the Pyranol characteristics. These are shown in Fig. 2.6. What is required is a combination of resistors and capacitors such that the frequency response of the combination approximates to these characteristics.

First of all series and parallel RC combinations are considered.

Series RC admittance, $Y = \dfrac{j\omega C(1-j\omega RC)}{1+\omega^2 R^2 C^2}$ 2.4a

Parallel RC admittance, $Y = \dfrac{1}{R} + j\omega C$ 2.4b

From equation 2.3 which expresses current in terms of voltage and complex permittivity.

* Pyranol 1476 (GEC, USA) A mixture of isomeric pentachlorodiphenyls

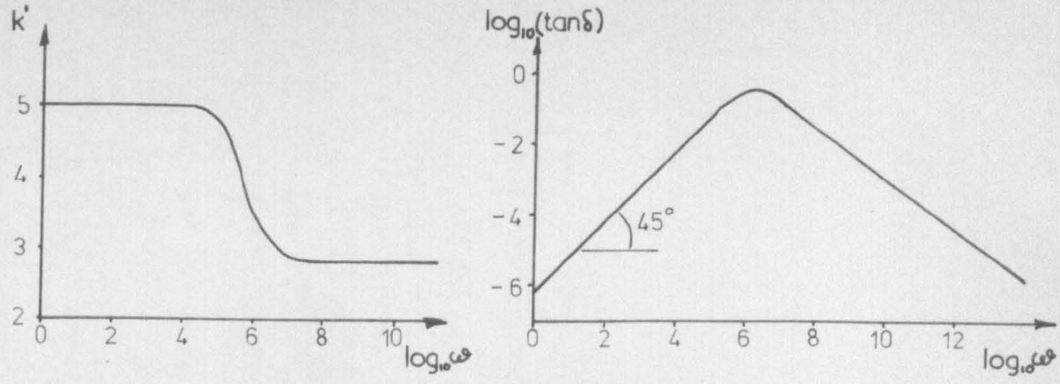


Fig.2.6 Characteristic of Pyranol, General Form

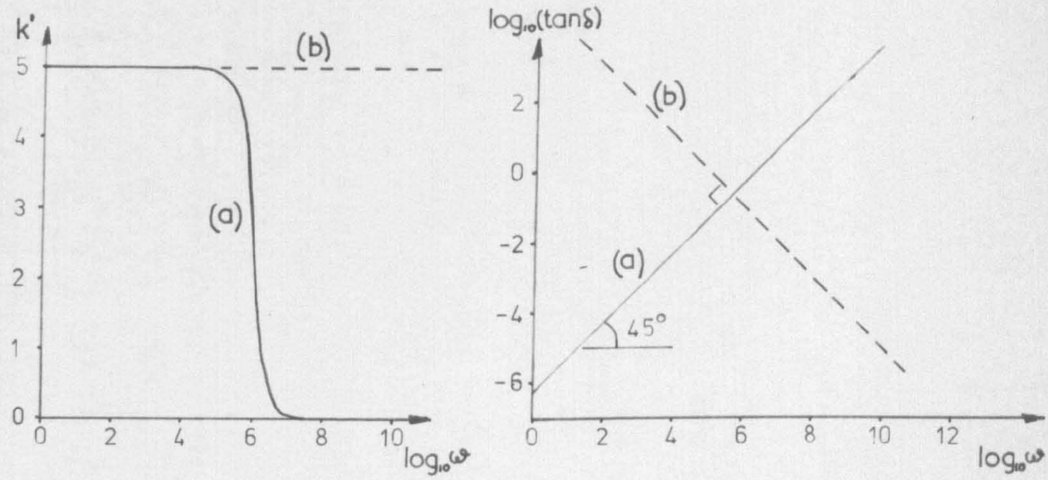


Fig.2.7 Characteristics of (a) series R-C (b) parallel R-C

Compare with Fig.2.6

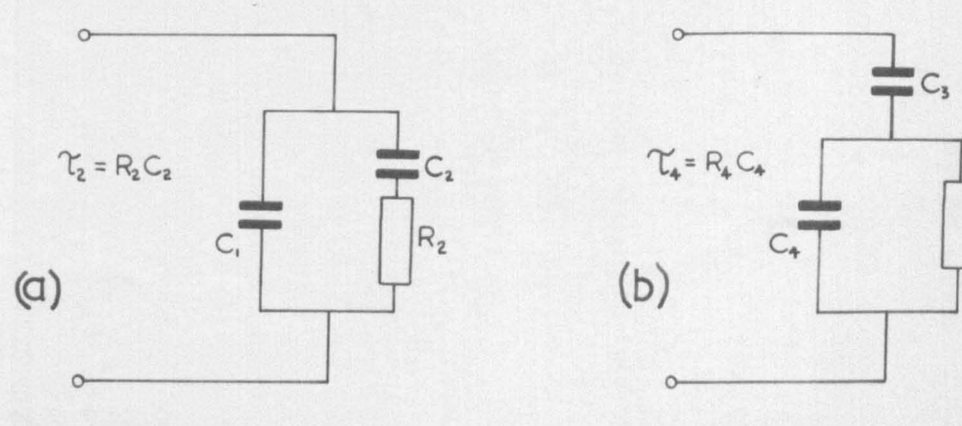


Fig.2.8 R-C Circuits Suitable for Representing

Dielectric Characteristics

$$Y = \omega C_o (k'' + jk') \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.5$$

The frequency response of k', k" and tan δ implied by the series and parallel R-C combinations can be obtained by comparing equations 2.4 and 2.5.

For series RC,

$$k' = \frac{1}{\omega C_o} \cdot \frac{\omega C}{(1 + \omega^2 R^2 C^2)}$$

$$k'' = \frac{1}{\omega C_o} \cdot \frac{\omega^2 R \; C^2}{(1 + \omega^2 R^2 C^2)}$$

$$\tan\delta = \frac{k''}{k'} = \omega R C$$

For parallel RC,

$$k' = \frac{1}{\omega C_o} \cdot \omega C$$

$$k'' = \frac{1}{\omega C_o} \cdot \frac{1}{R}$$

$$\tan\delta = \frac{k''}{k'} = \frac{1}{\omega R C}$$

The functions are plotted for comparison with Fig. 2.6, in Fig. 2.7. At low frequencies the series arrangement is a good approximation to the Pyranol characteristics in that the k' and the tan δ curves have the appropriate shapes. At high frequencies the series arrangement is a very poor approximation; the loss angle continues to increase with frequency and the combination appears to be almost purely resistive. The parallel RC combination is suitable where the series RC is unsuitable and the converse is also true. At any one particular frequency a series and a parallel arrangement of R and C can obviously be found to accurately represent the dielectric but if a range of frequencies is important a series-parallel circuit should be sought.

The combination of Fig. 2.8(a) gives :

$$Y = j\,\omega\,\frac{(C_1 + C_2 + j\omega\,\tau_2\,C_1)}{(1 + j\,\omega\,\tau_2)} \qquad \text{where } \tau_2 = R_2\,C_2$$

The frequency response of k' and tan δ are determined by comparing this equation with equation 2.5. This gives :

$$k' = \frac{C_1 + C_2 + \omega^2\,\tau_2^{\,2}\,C_1}{C_o\,(1 + \omega^2\,\tau_2^{\,2})} \quad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad 2.6$$

$$\text{and } \tan\delta = \frac{\omega\,\tau_2\,C_2}{(C_2 + C_1\,(1 + \omega^2\,\tau_2^{\,2}))} \quad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad 2.7$$

By sketching these functions it is clear that they are of similar form to the characteristics of Pyranol in Fig. 2.6. In order to show how good a fit can be achieved, a numerical example is given.

The capacitor geometry is assumed to be such that C_o = 0.2 μF since this means that C = 1 μF at low frequencies, if the dielectric is Pyranol. Values for the components in the circuit of Fig. 2.8 (a) are to be determined.

Equation 2.6 can be compared with values of k' taken from the actual characteristics of Pyranol in Fig. 2.9.

For large ω , $k' = \dfrac{C_1}{C_o} = 2.8 \quad \therefore \; C_1 = 0.56$ μF.

For small ω , $k' = \dfrac{C_1 + C_2}{C_o} = 5 \quad \therefore \; C_2 = 0.44$ μF.

Using equation 2.7 in a similar way

gives $\tau_2 = 22.8\,\mu s$

and $R_2 = \dfrac{\tau_2}{C_2} = 0.52\ \Omega$

The circuit of Fig. 2.8 (b) can be exactly equivalent to the circuit of Fig. 2.8 (a) i.e. the two circuits can be indistinguishable by two terminal measurements. The necessary and sufficient conditions for equivalence are obtainable from impedance equations

e.g. $$Z(\text{series}) = \frac{1 + s\tau_2}{sC_2\,(1 + \frac{C_1}{C_2} + sR_2\,C_1)},$$

$$Z(\text{parallel}) = \frac{1 + s(\tau_4 + R_4\,C_3)}{sC_3\,(1 + s\tau_4)}$$

The conditions to be met are then:

$$R_4\,C_3 + \tau_4 = \tau_2$$

$$C_3 = C_1 + C_2$$

$$C_3\,\tau_4 = C_1\,\tau_2$$

The appropriate values for these circuits when representing the 1 μF (nominal) Pyranol capacitor are shown in Fig. 2.9.

2.2.2.2.4 Capacitor with Resistive Connections and Plates and Real Dielectric

To represent capacitors which are real in every respect, the equivalent circuits of Fig. 2.4 and Fig. 2.8 should be combined This can be achieved by replacing each of the C/5 capacitors in Fig. 2.4 by say Fig. 2.8 (a) with R_2, C_1 and C_2 scaled by a factor of 5. The resultant circuit is rather complex. It is necessary to make further assumptions in order to obtain a circuit which is convenient to use.

The circuit must contain some series resistance because this is the only way of representing the series resistance of the connections. The rest of the circuit which corresponds to the dielectric characteristics and the plate resistance presents the problem. The plate resistance in power capacitors is really very small. This is because mechanical strength and cooling of the capacitor are important, consequently the plates are heavier and of more substantial construction than their counterparts used in electronics and light current work.

One method of obtaining a simplified circuit is to obtain k' vs ω and $\tan\delta$ vs ω characteristics for the capacitor, excluding connections. For power capacitors with their low plate resistance, these characteristics correspond closely with those of the dielectric and the equivalent circuit in Fig. 2.9 is then appropriate. From a knowledge of the capacitor's construction, an estimate of the connection resistance can be obtained and the equivalent circuit is then as shown in Fig. 2.10.

2.2.2.2.5 Discharging a Real Capacitor

The equivalent circuit for a real capacitor, derived in the preceding sections, is convenient for calculating capacitor discharge current and also for assessing the amount of energy dissipated within the dielectric. The component values in the circuit of Fig. 2.10 are typical and this 1 μF capacitor can be

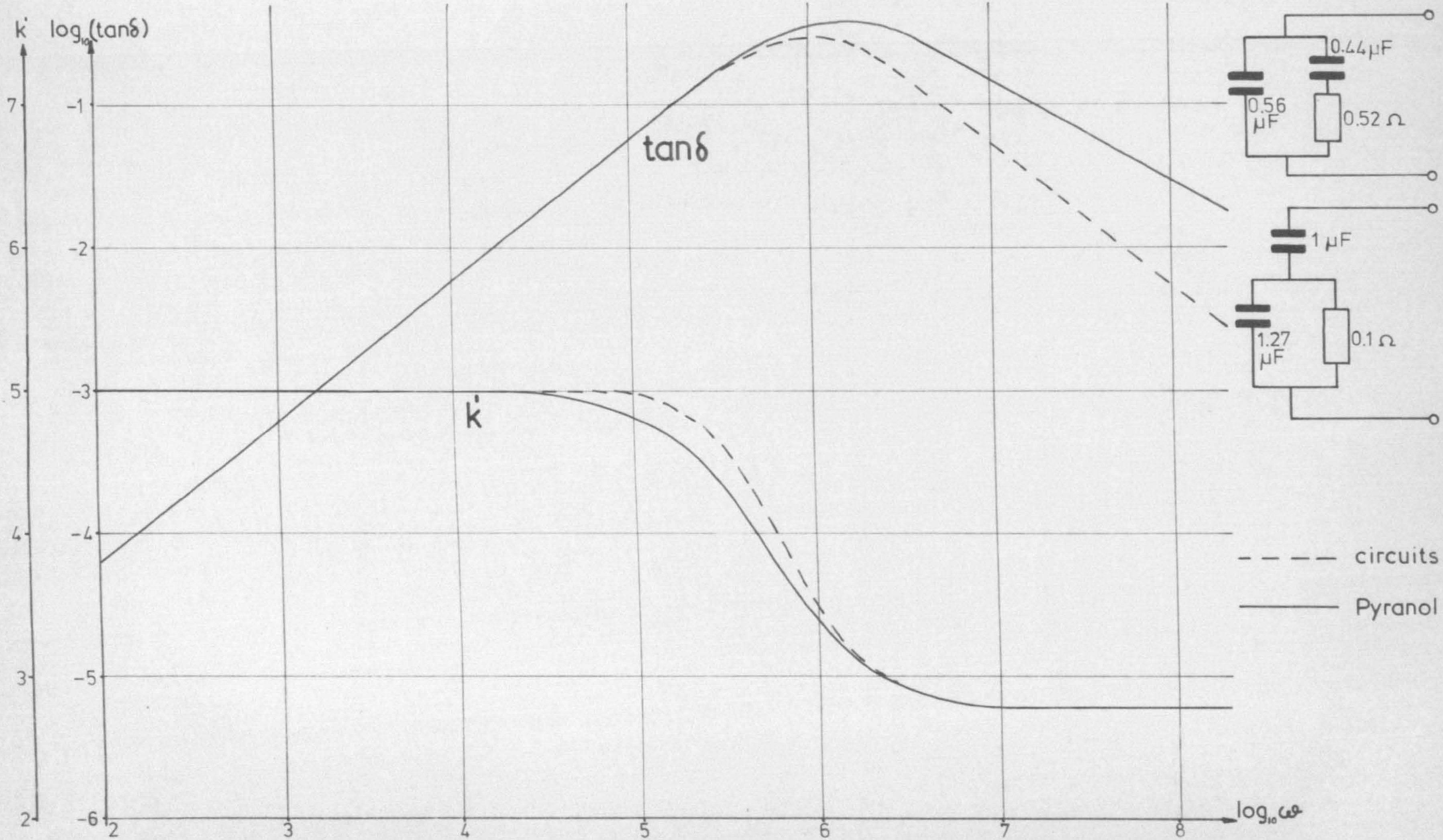


Fig.2.9 Characteristics of Pyranol compared with the Equivalent Circuits

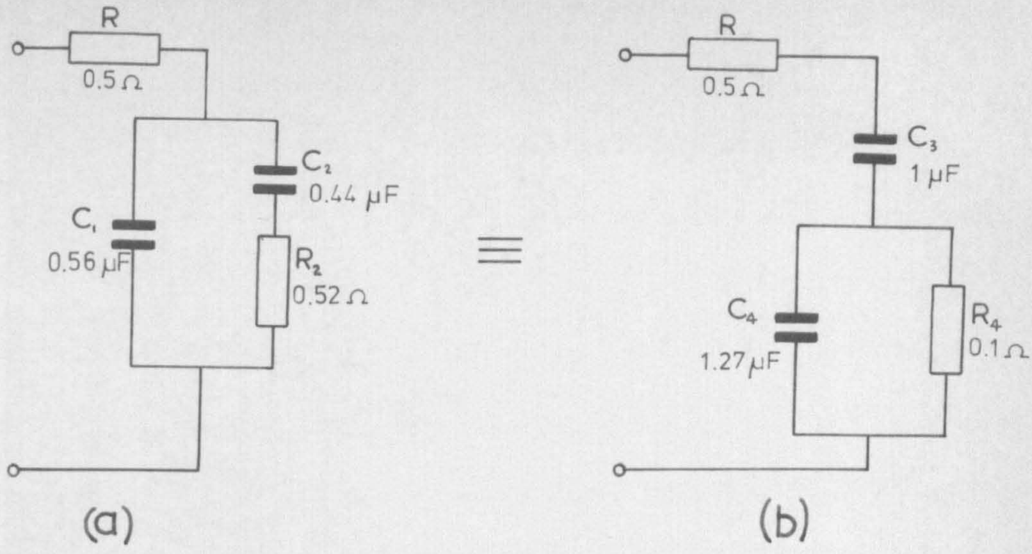


Fig. 2.10 Derived Equivalent Circuit of 1 μF Capacitor having Pyranol Dielectric

regarded as a scaled model of the much larger series capacitor. If the capacitor is charged and then it is suddenly short circuited, 40 per cent of the energy in the fully charged capacitor is dissipated in the dielectric. This figure is calculated in Appendix 2.2. If series capacitors are so treated and the charging and shorting cycle is repeated several times in quick succession, then it is reasonable to expect that the dielectric will become overheated and damage could be caused. This type of cycling occurs when a large fault current flows in a transmission line fitted with a series capacitor and a self extinguishing protective spark gap. Typically, five or six charge and shorting cycles occur if the fault is cleared quickly but there are many more of these cycles if the fault is cleared after a delay e.g. in Zone 2 time. The proportion of energy dissipated in the dielectric depends to a large extent on the impedance of the discharge circuit. The 40 per cent figure is for discharge through zero impedance. If the discharge impedance is 100 ohms, not an unreasonable figure bearing in mind the value of 1 μF, then there would be about 0.1 per cent of the capacitor energy dissipated in the dielectric. This represents at least 100 times the energy dissipation associated with normal capacitor operation.

It is reasonable to conclude that a capacitor which is protected by an extinguishing type of spark gap will have a shorter life expectancy than the capacitor which is protected by a simple gap. This is because the former is subjected to greater thermal stresses within its dielectric and the latter is allowed to rest after having been discharged once. The mechanisms of eventual failure are many. Some are listed below :

1. Effect of impurities in the dielectric.
2. Breakdown of polymer chains due to dipole rotation.
3. Breakdown due to free charges in voids in the material

A quantitative assessment of the life expectancy of a capacitor subject to a given duty cycle is difficult, if not impossible, and the subject is, therefore, pursued no further.

2.3. Sources of Energy

In normal circumstances, the sources of energy in a power system are the generating stations. Mechanical power drives synchronous machines which supply power to the transmission system via transformers and switchgear. In the analysis presented in Chapter 3, these energy sources are represented and some mention of these items is appropriate at this stage.

For simplicity it is usually desirable to use an equivalent source of energy at a particular point in a power system i.e. use a Thevenin generator. In this way the difficulties associated with the analysis of a complex of generating stations, transformers and transmission lines are eliminated and attention can be focussed on a particular item e.g. a transmission line. In this way problems are solved much more easily but with some reduction in accuracy. The purpose of the discussion which follows is to show that the form of the equivalent circuit used is a reasonable approximation to the truth.

2.3.1. Electrical Characteristics

In the analysis presented in Chapter 3, the generated E.M.F. is imposed on the transmission line via a source impedance which is inductive. Its value represents the so called Synchronous Inductance of the alternators in series with other impedance elements, mainly the generator transformer. The transformer impedance is essentially its leakage inductance and the synchronous inductance is the sum of armature leakage inductance and an inductance associated with armature reaction. The basic assumption that the magnetisation curve is the air gap line is well justified when, under fault conditions, the machine delivers a large lagging current and the flux is considerably reduced by armature reaction.

Under transient conditions e.g. when the output terminals of a machine are suddenly short circuited, the current that flows cannot be accurately determined by making use of the synchronous inductance and generalised machine equations need to be solved. The components of the current that flows during such a disturbance can be separated into unidirectional, steady state, transient and sub-transient components. If the circuit to which the synchronous machine is connected is fairly simple, then the equations for these components can be written down. However, for circuits such as long transmission lines, the task of solving the appropriate equations is exceedingly complicated. A further complication is due to the voltage regulator on the machine which attempts to maintain the terminal voltage constant. Voltage regulators do not respond instantaneously. The calculation of the exact value of current is, therefore, a laborious process and it varies with the excitation system used. A step-by-step method is given in ref. 22 p.166.

Because of these difficulties, it is not usual to consider the machine equations in power system studies. Fault currents depend, in general, much more on transmission line impedance. It is only when actual machine currents are of particular interest that the synchronous machines are represented in more detail.

2.3.2 Mechanical Characteristics

In the stability studies in Chapter 3, it is the mechanical stability of the turbo-alternator sets which is of interest. In this respect, machines operating in parallel behave as a single unit and, therefore, a consideration of a single turbo-alternator can be easily extended to a number of sets. A knowledge of the inertia constant, H, is required in the determination of the acceleration and deceleration of the rotating parts of a turbo-alternator. This constant represents the stored energy per kVA and can be computed from the moment of inertia and synchronous speed. For large turbo-alternators, the value of H[22] lies between 2 and 4 and the inertia of a particular source of energy can therefore be calculated. The mechanical power input to the system must also be known for stability analysis; this is the quantity of steam input to the turbines. In a short period immediately after a fault, this steam input is constant because there is a time lag in response of the governor/steam regulator system. For periods greater than about 0.5 seconds, it is necessary to take into account this factor if accurate studies are to be achieved.

2.3.3. Switchgear Performance

The performance of the switchgear which connects sources of energy to transmission lines is important because it affects the time for which system faults are present.

The operation of high speed circuit breakers is taken into account in the stability analysis in Chapter 3 where it is assumed that they operate in approximately 5 cycles i.e. 100 ms. This time is, in practice, made up of 2 cycles for fault detection and 3 cycles for breaker operation. It is sufficient to consider the breaker as three separate switches, one in each phase of each transmission line, and these may be considered to open at the first zero crossings of each of the phase currents following the 5 cycle delay. Operation time is virtually independent of current magnitude.

3. POWER SYSTEM ANALYSIS

Of the many aspects of power system analysis, it is the particular analytical problems associated with series compensated power lines which are given attention in this chapter.

Harmonic and sub-harmonic resonance are dealt with first, and an analytical approach is proposed which is helpful in understanding how sub-harmonics can arise. In section 3.2, the transient stability of a two machine system is analysed and the results aimed at identifying the advantages of extinguishing gaps fitted to the series capacitors. In section 3.3, long line equations are used in the calculation of travelling waves on power lines during fault conditions. The method of analysis is used as a tool in the evaluation of distance protection schemes in Chapter 4.

3.1 Harmonic and Sub-harmonic Resonance

These phenomena caused problems on the early series compensated lines and were used in arguments against the application of series capacitors.

Harmonic resonance, often called ferro-resonance, occurs in some non-linear circuits when the amplitude or frequency of the exciting voltage is incremented at one of the critical values where harmonic resonance occurs. The phenomenon is, therefore, an example of jump resonance. A capacitor and some form of non-linear inductance, usually associated with transformer magnetisation, are necessary for the condition to arise. During the resonance, strong odd harmonics are present in the line current and consequently high voltages build up on the series capacitors. Resistors, strategically placed, are effective in preventing harmonic resonance in most cases, but power system configurations do occasionally result in the sufficient conditions being met and then the simplest remedy is to short circuit the series capacitor. The circumstances under which harmonic resonance arises have been examined by Swift.[25] In an example, he shows that the current in a particular circuit increases in amplitude by a factor of 10 when the exciting sinusoidal voltage is changed by only 1%.

Sub-harmonic resonance involves energy transfer from fundamental to a sub-multiple frequency, usually 1/3 or 1/5. Unlike harmonic resonance, sub-harmonics do not necessarily arise even if suitable excitation and circuit parameters are chosen. The effect is not, therefore, a form of jump resonance but must be initiated by an appropriate sudden change in the parameters of the system. Sub-harmonics are of particular interest in series compensated transmission circuits because of the associated high voltages across the capacitors. Some attention was given to the matter by Butler and Concordia[12] and others.[26] Renewed interest has been shown by a number of researchers more recently.[27,28] The author has presented further analysis which, it was felt, was needed to lead to a fuller understanding of the behaviour. The content of this section is substantially a copy of this paper.[13]

First of all, the steady state conditions are described.

Secondly, consideration is given to one way in which a circuit operating in a particular mode may be caused to change to a different mode which will persist until some further happening causes another change.

3.1.1 Linear and Non-linear Circuits

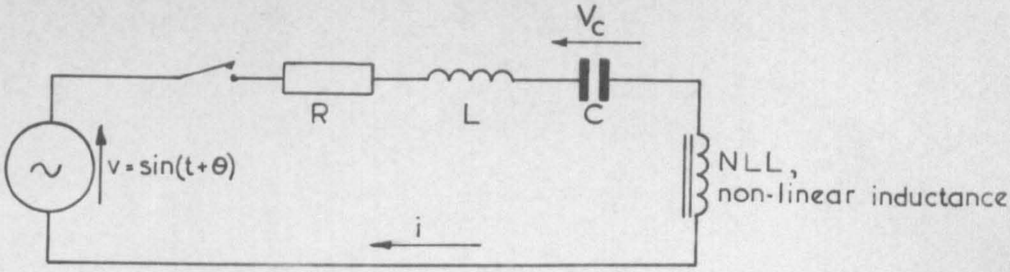
The current which flows in a linear circuit, after some change of conditions takes place, consists of a transient component and a steady-state component which is independent of the conditions prior to or at the instant of change. In the analysis of such a circuit the solution of the differential equations gives a complementary function (transient term) and particular integral (steady-state term). The latter is not dependent on the initial conditions.

With a non-linear network the above situation does not arise and the steady-state behaviour may well depend on the conditions at the time of a disturbance or initial closure of the circuit. This is illustrated below, firstly for a loss-free circuit and then the effect of resistance is taken into account.

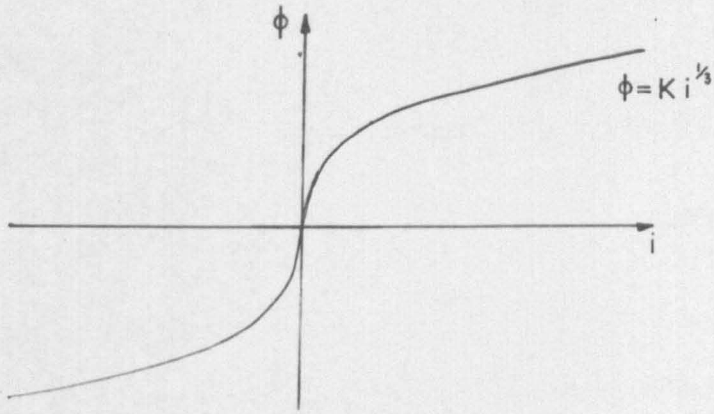
3.1.1.1 Behaviour of a Loss-free, Non-linear Circuit

For simplicity the circuit shown in Fig. 3.1(a) with R=0 was analysed, the non-linear inductor (NLL) being assumed to have a cubic relationship between flux and current as shown in Fig. 3.1(b).

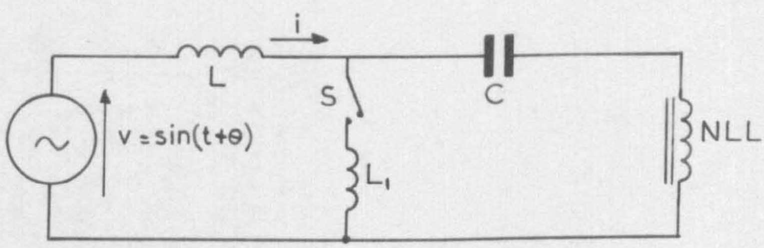
For this circuit:



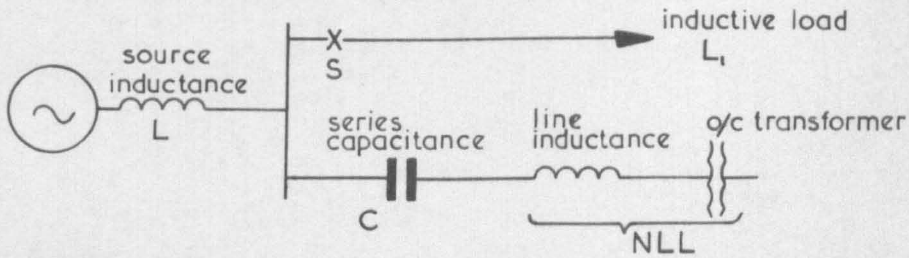
(a) non-linear circuit studied



(b) approximated magnetisation for non-linear inductance



(c) non-linear circuit representing (d), below



(d) section of a power system

Fig.3.1 Relating to Sub-harmonic Resonance Analysis

$$L \frac{di}{dt} + \frac{1}{C} \int_0^t i \, dt + V_c(0) + K \cdot \frac{d(i^{1/3})}{dt} = \sin(t + \theta) \quad .. \; 3.1$$

One possible solution of this equation is :

$$i = A^3 \cos^3\left(\frac{t + \theta}{3}\right) = \frac{A^3}{4} \cdot \left(3 \cos\left(\frac{t + \theta}{3}\right) + \cos\,(t + \theta)\right)$$

$$\text{where } 4K = 3A^2\left(\frac{9}{C} - L\right) \quad .... \; 3.2$$

$$\text{and} \quad 4 = A^3 \left(\frac{1}{C} - L\right) \quad .... \; 3.3$$

Another solution is:

$$i = B^3 \cos^3(t + \theta) = \frac{B^3}{4} (3 \cos(t + \theta) + \cos 3(t + \theta))$$

$$\text{where } 1 + KB = \tfrac{3}{4} B^3 \left(\frac{1}{C} - L\right) \quad .... \; 3.4$$

$$\text{and} \quad 3L = \frac{1}{3C} \quad .... \; 3.5$$

A circuit containing any set of values of the L, C and K parameters which satisfy equations 3.2 - 3.5 will clearly have two possible modes of operation, one of which will have harmonics in its current and the other sub-harmonics. Such a set of values is $L = 0.1H$, $C = \frac{10}{9}$ F and $K = 17.5$ which correspond in their relative magnitudes to those in power systems incorporating series capacitors and these give possible currents of:

$$i = 3.75 \cos\left(\frac{t + \theta}{3}\right) + 1.25 \cos(t + \theta)$$

$$\text{and} \quad i = -10^{-4} (1.39 \cos(t + \theta) + 0.462 \cos 3(t + \theta))$$

If θ is chosen to be $3\pi/2$ then the initial conditions are :

$i(0) = 0$, $V_c(0) = \frac{KA}{3} - 1 = 9$ for the first condition

and

$i(0) = 0$, $V_c(0) = -1 - KB = -1.1 \times 10^{-3}$ for the second.

This indicates that if the circuit was energised at a negative voltage peak and the capacitor was charged to 9 V then a sub-harmonic current component would flow, whereas if the initial voltage on the capacitor had been -1.1 mV, a harmonic current component would be present. For other instants of energisation different currents would be obtained.

3.1.1.2 Behaviour of a non-linear circuit containing resistance

For the circuit shown in Figure 3.1(a) when the resistance is not neglected:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\cdot\int_0^t i\,dt + V_c(0) + K\frac{d(i^{1/3})}{dt} = \sin(t+\theta) \qquad \ldots \quad \ldots \quad 3.6$$

This equation is difficult, if not impossible, to solve by classical means and, therefore, a method using a digital computer was adopted. It produced, as a solution, values of the current (i) for a series of instants of time (t) and this enabled the time variation of the current to be plotted for a given set of component values and initial conditions. The process must be repeated each time the effect of a new component value or condition is to be examined.

For computational purposes equation 3.6 was rewritten as:

$$\frac{d^2q}{dt^2}\cdot\left(L + \frac{K}{3}\left(\frac{dq}{dt}\right)^{-2/3}\right) + \frac{dq}{dt}R + \frac{1}{C}q = \sin(t+\theta)$$

where q = charge on the capacitor.

The Kutta-Merson initial value formulae[29] can be readily applied to this equation and by adjusting the step length associated with the computation, the accuracy of the solution can be controlled.

Two sets of computed results for the circuit shown in Fig. 3.1(a) are shown in Figures 3.2 and 3.3, which are based on values of R = 0.1 and 0.05 respectively, the other parameters in each case being L = 0.1H, C = 1.11F, K = 17.5, $\Theta = 1.5\pi$ and $V_c(0)$ = 9 V. The supply voltage variation is shown in both figures and for comparison the current which could be obtained under loss-free conditions is also included. Detail of the current waveforms in Fig. 3.2 are shown in Fig. 3.4.

The sub-harmonic current component is clearly transient for the conditions represented in Figure 3.2. With the reduced resistance used for Figure 3.2, the sub-harmonic behaviour persists into steady state.

3.1.2. Transition Between Operating Modes

It is clearly of interest to show that circuit operation may be caused to change from one mode to another.

The circuit shown in Fig. 3.1(c) represents the power system illustrated in Fig. 3.1(d).

When the switch S is open, equation 3.1 is applicable and when it is closed, the following equation, which is similar in form, applies:

$$L\frac{di}{dt} + \frac{(L+L_1)}{L} \cdot \frac{1}{C} \cdot \int_0^t i\ dt + \frac{(L+L_1)}{L_1} \cdot V_c(0) + \frac{(L+L_1)}{L_1} \cdot K\frac{d(i^{1/3})}{dt} = \sin(t+\Theta) \qquad \ldots\ \ldots\ \ldots\ 3.7$$

If the system were operating with the switch open, then a current given by $i = B^3\cos^3(t+\Theta)$ could, as shown in section 3.1.1.2 be flowing. Should the switch S then be closed at a

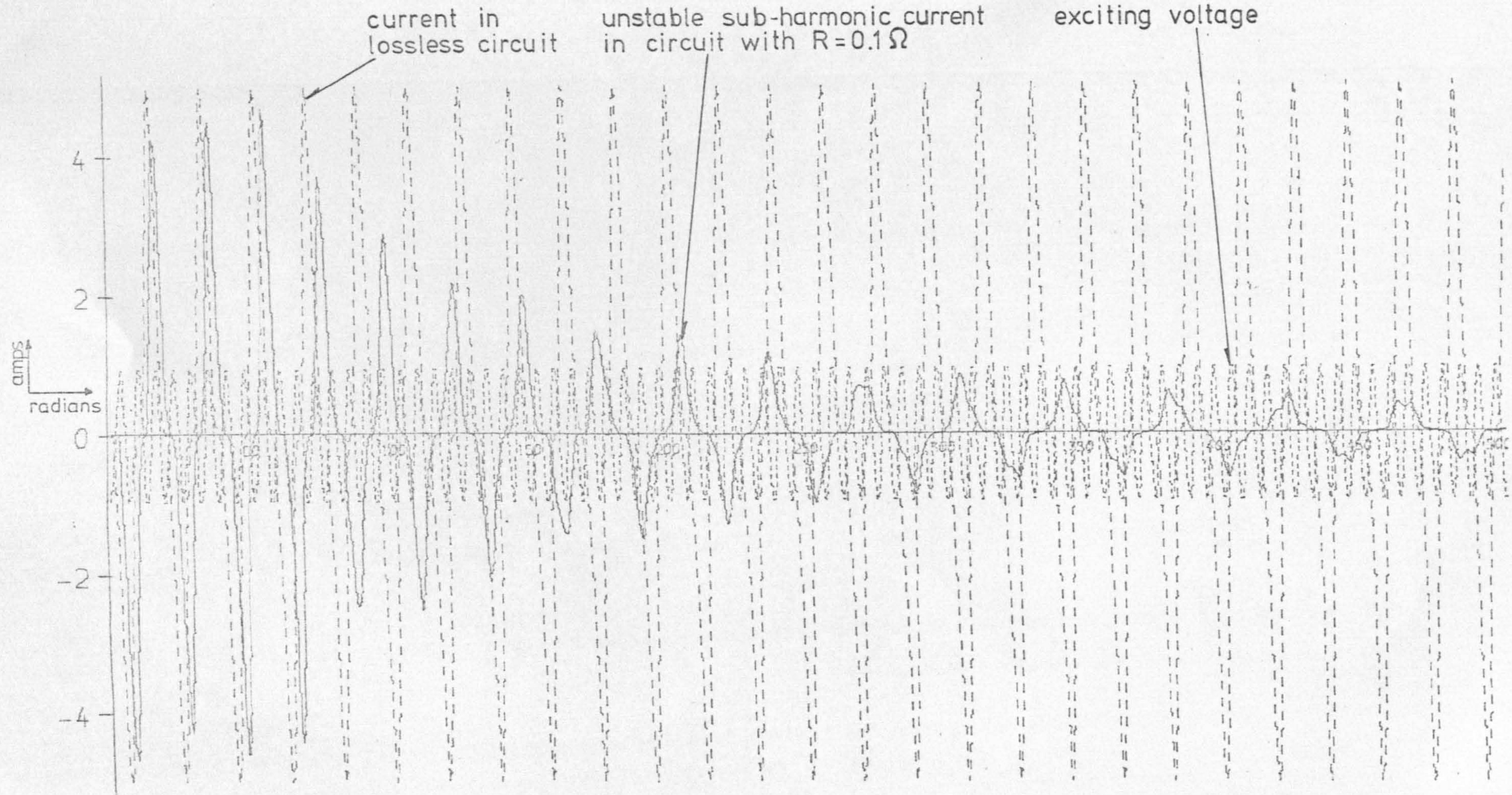


Fig.3.2 Computed Wave-forms Showing Instability of Sub-harmonics

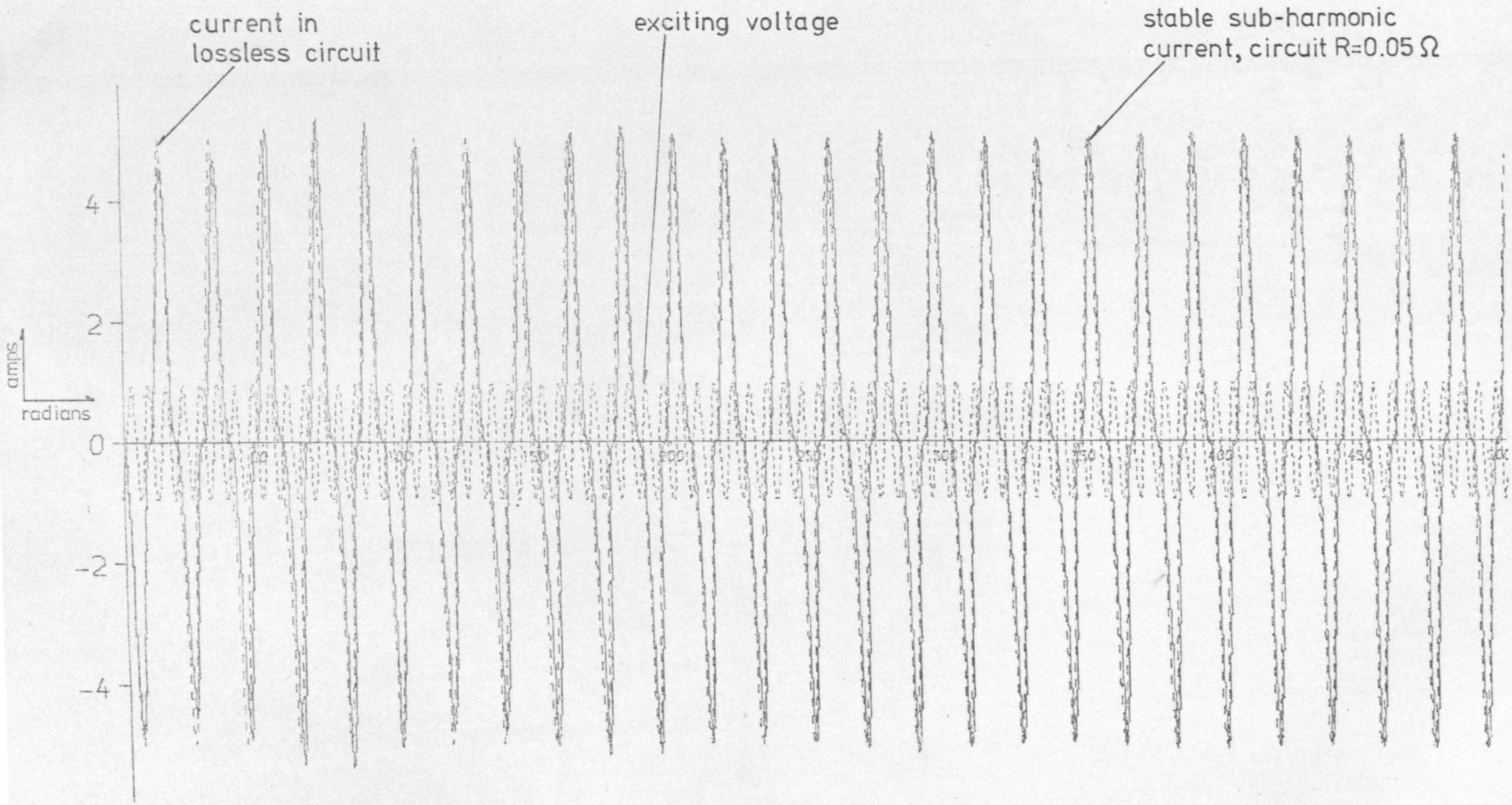
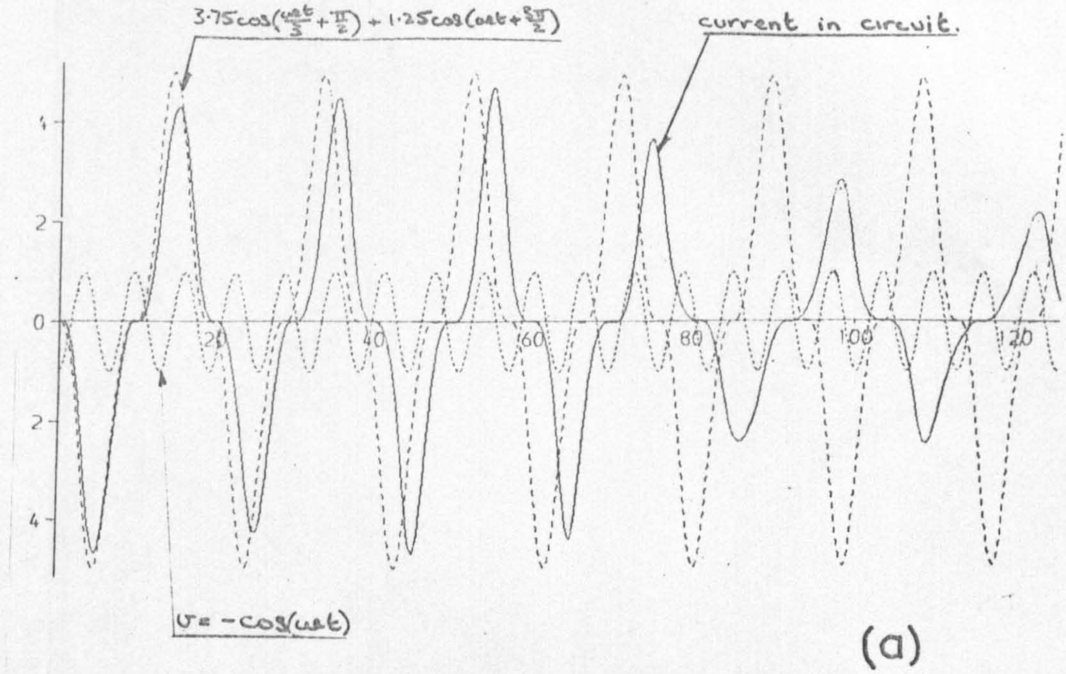


Fig.3.3 Computed Wave-forms Showing Stable Sub-harmonics



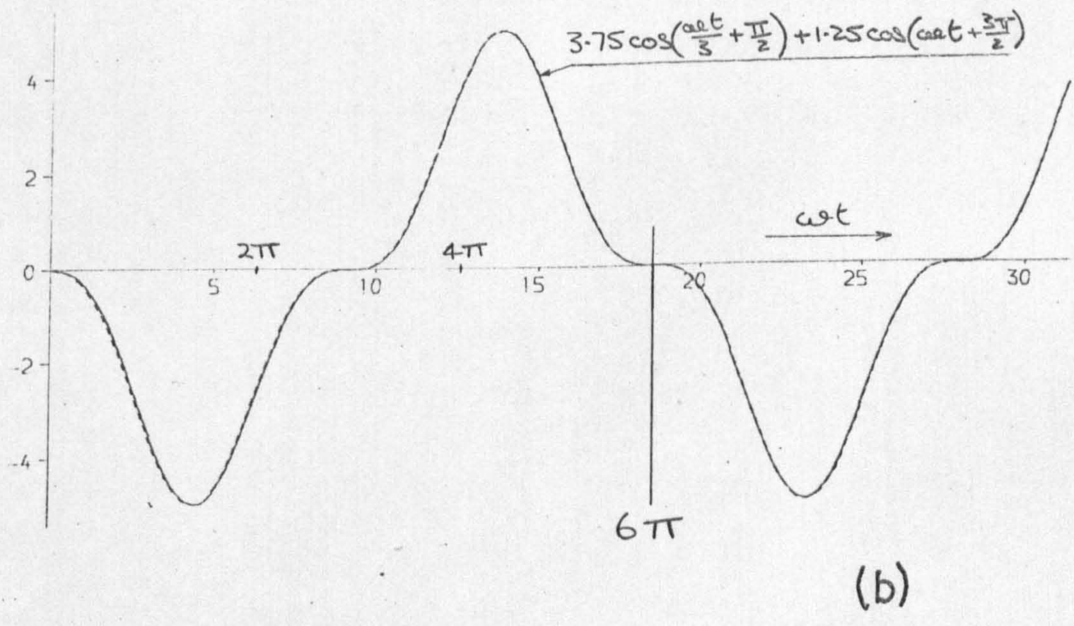


Fig. 3.4 Detail of Fig.3.2 in (a),
(b) Comparison of Calculated and Analytical Wave-forms for Current

negative voltage peak; i.e. $\Theta = 3\pi/2$, time being measured from this instant, the current would become :

$$i = D^3\cos^3\left(\frac{t+\Theta}{3}\right)$$

if the capacitor voltage at the instant of switching was given by

$$V_c(0) = -1 - KB \qquad \ldots \ldots \ldots \quad 3.8$$

where

$$4aK = 3D^2\left(\frac{9a}{C} - L\right) \qquad \ldots \ldots \ldots \quad 3.9$$

$$4 = D^3\left(\frac{a}{C} - L\right) \qquad \ldots \ldots \ldots \quad 3.10$$

$$aV_c(0) = aK\,\frac{D}{3} - 1 \qquad \ldots \ldots \ldots \quad 3.11$$

and

$$a = \frac{L + L_1}{L_1} \qquad \ldots \ldots \ldots \quad 3.12$$

A circuit containing any set of values which satisfy equations 3.4, 3.5 and 3.8 - 3.12 simultaneously would thus change from a mode in which the current contained harmonics to one with sub-harmonics if the switch was closed at a negative peak of the supply voltage.

One such set of values is $L = 100$ mH, $L_1 = 3.3$ mH, $C = \frac{10}{9}$ F and $K = 1.66$ for which a current of :

$i = -0.569\cos(t+\Theta) - 0.190\cos 3(t+\Theta)$ will flow before switch closure, t being negative, and

$i = 0.393\cos\left(\frac{t+\Theta}{3}\right) + 0.131\cos(t+\Theta)$ will flow after switch closure at $t = 0$ when $\Theta = 3\pi/2$.

The above illustrates one way in which a transition to sub-harmonic resonance may occur. Only direct change from one steady state condition to another has been considered but there will certainly be situations where transient current components are present during the transition leading to a more gradual change.

3.2 Transient Stability Study

The purpose of undertaking the work described in this section was firstly to develop a method of achieving the transient stability analysis of a power system containing series capacitors, and secondly to observe the effect of two of the different types of protective spark gap which are fitted to series capacitors. The characteristics of the different spark gaps have been described in section 2.2.1. It is thought that extinguishing gaps are better than simple gaps when a transient stability problem exists.[30]

3.2.1 Analysis

The simple two machine system of Fig. 3.5 was chosen for study because it is a reasonable approximation to a real power system and it is sufficient to enable a comparison to be made between the two types of protective spark gap. The following sequence of events was studied.

a. A system running in steady-state with no fault present and constant angle of separation between the two machines. This means that one machine must generate and the other must motor.

b. A single phase line to earth fault occurs on circuit 1 and after a typical fault detection and clearance time, c.f.

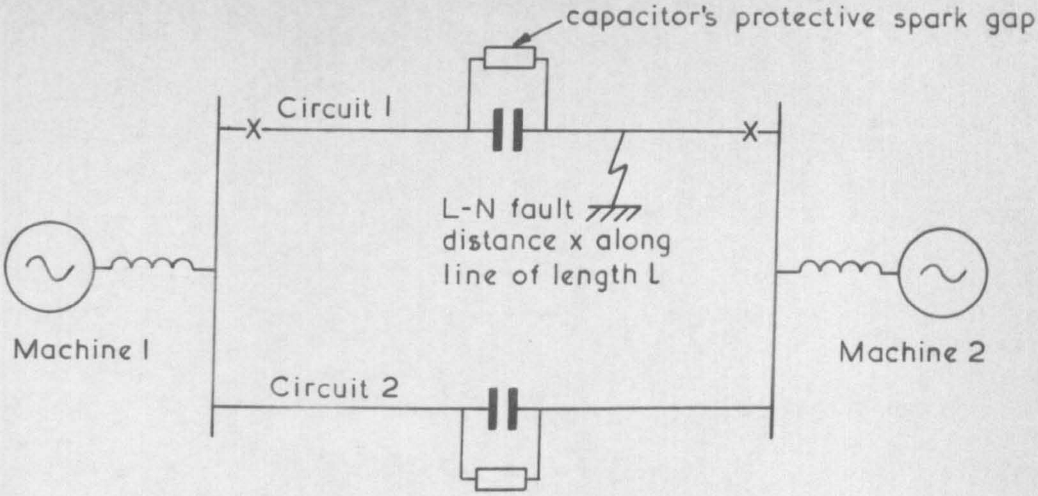


Fig.3.5 System Studied in Stability Analysis

section 2.3.3, the fault is cleared by the opening of the three phase breakers at each end of the faulted circuit.

c. The change in the machine angles throughout periods a and b and also in the period after clearing the fault was determined.

With this sequence of events, the transient stability problem is started by applying the single phase fault but it is made worse by switching out the faulty circuit 1. The common line to earth fault, characterised by its high current, was chosen because of its effect on the series capacitor.

It was considered sufficient to represent the protective spark gaps by equivalent switches. A simple gap was represented by a switch which short-circuits the capacitor when its voltage reaches the setting of the spark gap and remains closed for a period of at least one second. An extinguishing gap was represented by a switch which short-circuits the capacitor, as with the simple gap, and then opens again to reinsert the capacitor when the current next passes through zero. This rapid extinction of the gap current is a characteristic of the magnetically quenched gaps described in section 2.2.1.2.

As is usual in stability studies, line susceptance was neglected and each source impedance was represented by a linear inductance. The circuit representation of the system studied is thus as shown in Fig. 3.6.

If it is assumed that the machines are undamped then

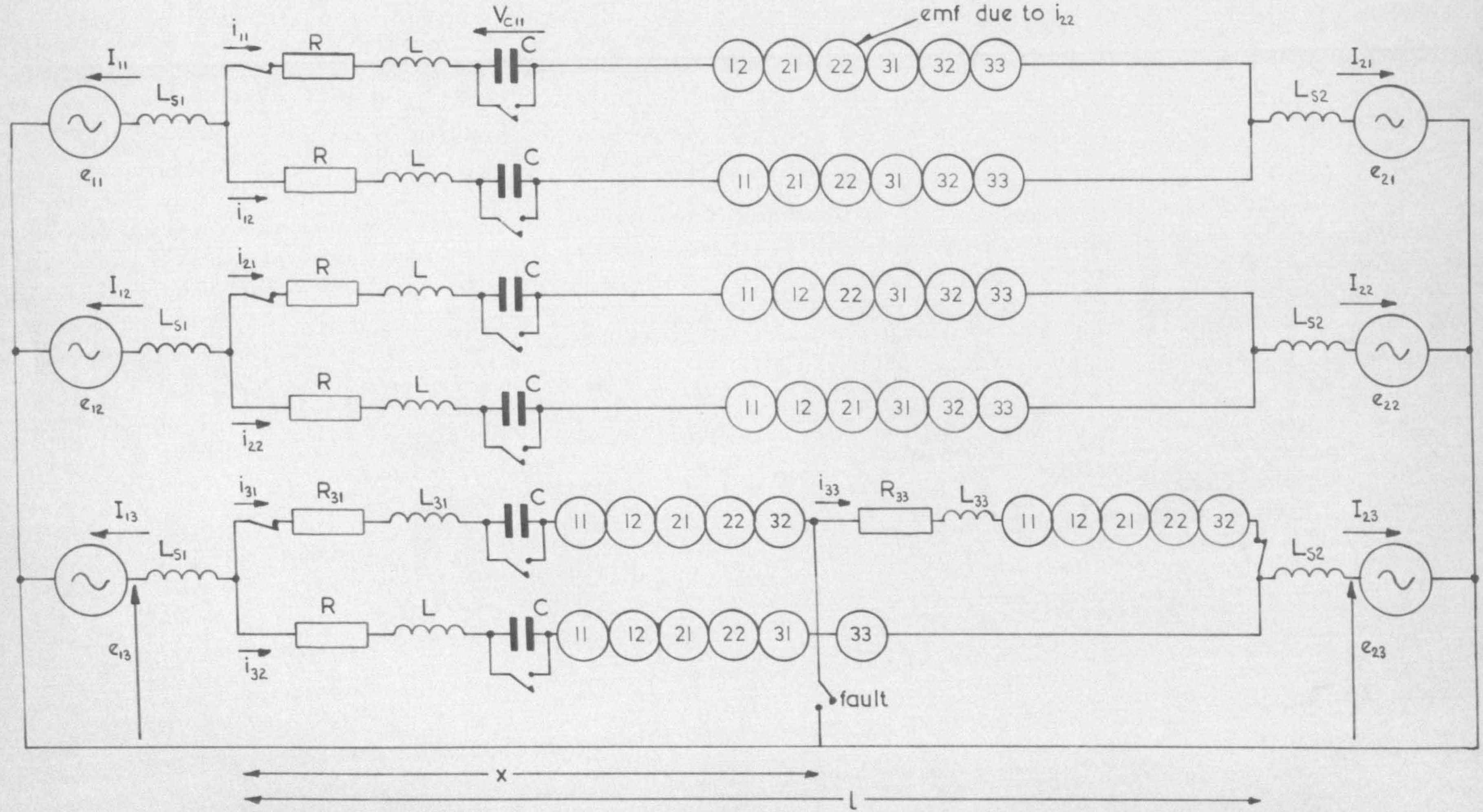


Fig.3.6 Circuit Representation of the System Studied (see Fig.3.5)

the mechanical stability depends only on the mechanical power fed in (e.g. steam) and the electrical power generated. The appropriate equation is then

$$p^2A = J^{-1} \times (P_{mech} + P_{elec}) \qquad \text{.. .. ..} \quad 3.13$$

The elements of the column matrix i.e. A_1 and A_2, define the angles by which the machines are in advance of a reference machine which is rotating at angular speed, ω. J^{-1} is a diagonal matrix depending on machine inertias (radians/sec/joule). If it is desired to take account of steam governors, then the elements of P_{mech} vary with time. In the analysis, these elements were held constant. The elements of P_{elec} represent the instantaneous power flowing into the machine so that

$$P_{elec_1} = I_{11}(t).E_{11}(t,A_1)+I_{12}(t).E_{12}(t,A_1)+I_{13}(t).E_{13}(t,A_1) \qquad \text{.. .. ..} \quad 3.14a$$

$$P_{elec_2} = I_{21}(t).E_{21}(t,A_2)+I_{22}(t).E_{22}(t,A_2)+I_{23}(t).E_{23}(t,A_2) \qquad \text{.. .. ..} \quad 3.14b$$

In these equations, the elements, E_{11}, E_{12} ..., are the E.M.F.s of the two machines. More precisely they are :

$$E_{jk} = E_{mj} \operatorname{Sin}(\omega t + \alpha + \frac{2k\pi}{3} + A_j) \qquad \text{.. .. ..} \quad 3.15$$

where $k = 1, 2, 3.$

$j = 1, 2.$

α = an arbitrary constant.

The values of E_{m1} and E_{m2} in the above equation do, in practice, vary with time due to the action of the excitation control,but for simplicity these were assumed to be constant.

The only other functions which need to be expressed in order that equations 3.14 and hence equation 3.13 can be solved are the instantaneous machine currents I_{11}, I_{12} .... They depend on the circuit shown in Fig. 3.6. This circuit is described by the equation.

$$(L+M) \times p^2 I + R \times pI + C^{-1} \times I = pE$$

The elements in the coefficient matrices are determined by applying Kirchoff's Laws to the circuit. The matrices which should be read in conjunction with Fig. 3.6, are written out in full.

$$I = \begin{bmatrix} i_{11} \\ i_{12} \\ i_{21} \\ i_{22} \\ i_{32} \\ i_{32} \\ i_{33} \end{bmatrix} \qquad E = \begin{bmatrix} E_{11} - E_{21} \\ E_{11} - E_{21} \\ E_{12} - E_{22} \\ E_{12} - E_{22} \\ E_{13} - 0 \\ E_{13} - E_{23} \\ 0 - E_{23} \end{bmatrix} \qquad V_c = \begin{bmatrix} V_{c11} \\ V_{c12} \\ V_{c21} \\ V_{c22} \\ V_{c31} \\ V_{c32} \\ ? \end{bmatrix}$$

$$L = \begin{bmatrix} L+L_{s1}+L_{s2} & L_{s1}+L_{s2} & 0 & 0 & 0 & 0 & 0 \\ L_{s1}+L_{s2} & L+L_{s1}+L_{s2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L+L_{s1}+L_{s2} & L_{s1}+L_{s2} & 0 & 0 & 0 \\ 0 & 0 & L_{s1}+L_{s2} & L+L_{s1}+L_{s2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{31}+L_{s1} & L_{s1} & 0 \\ 0 & 0 & 0 & 0 & L_{s1} & L+L_{s1}+L_{s2} & L_{s2} \\ 0 & 0 & 0 & 0 & 0 & L_{s2} & L_{33}+L_{s2} \end{bmatrix}$$

$$C = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \infty \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & M_2 & M_1 & M_2 & M_{311} & M_2 & M_{331} \\ M_2 & 0 & M_2 & M_1 & M_{312} & M_1 & M_{332} \\ M_1 & M_2 & 0 & M_2 & M_{311} & M_2 & M_{331} \\ M_2 & M_1 & M_2 & 0 & M_{312} & M_1 & M_{332} \\ M_{311} & M_{312} & M_{311} & M_{312} & 0 & M_{312} & 0 \\ M_2 & M_1 & M_2 & M_1 & M_{312} & 0 & M_{332} \\ M_{331} & M_{332} & M_{331} & M_{332} & 0 & M_{332} & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} R & R_2 & R_1 & R_2 & R_{311} & R_2 & R_{331} \\ R_2 & R & R_2 & R_1 & R_{312} & R_1 & R_{332} \\ R_1 & R_2 & R & R_2 & R_{311} & R_2 & R_{331} \\ R_2 & R_1 & R_2 & R & R_{312} & R_1 & R_{332} \\ R_{311} & R_{312} & R_{311} & R_{312} & R_{31} & R_{312} & 0 \\ R_2 & R_1 & R_2 & R_1 & R_{312} & R & R_{332} \\ R_{331} & R_{332} & R_{331} & R_{332} & 0 & R_{332} & R_{33} \end{bmatrix}$$

In these equations M_1 and R_1, M_2 and R_2 represent the interphase and the intercircuit coupling respectively. In the faulted phase, the parameters which vary with line length, namely L, R, M_1, M_2, R_1 and R_2, are divided according to fault position thus

$$L_{31} = \frac{x}{l} \cdot L; \; M_{332} = (1 - \frac{x}{l}).M_2 \text{ etc.}$$

The driving voltages in the circuit have changing frequency due to the changing values of A_1 and A_2. An analytical treatment of the equation is therefore difficult, if not impossible, and numerical techniques were chosen instead. The initial value technique of Runge Kutta[29] proved to be suitable for solving the equations on a digital computer.

During the course of obtaining a solution, the capacitor voltages were observed by using the equation:

$$V_c = E - R \times I - (L + M) \times pI$$

If any of the capacitor voltages exceeded the threshold level of the spark gap then initial conditions were reset by using the equation :

$$pI = (L+M)^{-1} \times (E - V_c - R \times I)$$

Also the appropriate element in the C^{-1} matrix was set to zero in order to represent the circuit with the appropriate spark gap operating. Capacitors were allowed to function again, if the capacitor protection was an extinguishing gap, by first of all detecting a zero crossing in the gap current and then by reinstating the appropriate element in the C^{-1} matrix.

The opening of the line circuit breakers were modelled by detecting current zero crossing in the appropriate phase and then resetting initial conditions.

3.2.2. Results

3.2.2.1 Checking of Results

It is always desirable to verify computed results by comparing them with other data such as experimental or analytical results. Checking against experimental data was not possible because the computer program does not fully model the synchronous machines. It was decided to check the computed results by assessing the amount of power issuing from the two machines, the amount of mechanical power associated with each machine and hence the amount of disturbing power available for accelerating each machine. In this way the computed rotor angle swing curves were checked against calculated values for particular wave-forms of currents and voltages at the machine terminals. The analytical checking of the actual currents and voltages produced by the computer was difficult to achieve because of the changing frequency of the machine E.M.F.s. However, by making the machine inertias very large, computer results were obtained for the constant frequency case and this enabled computed currents and voltages to be checked against calculated waveforms.

3.2.2.2 Effect of Mutual Coupling and Source Impedance

There is always mutual coupling between phases of transmission lines; however, it is instructive to compare the behaviour of lines which have and have not, such coupling. If the currents in a line are separated in phase by about 120° but differ in amplitude due to either different load/phase or else unequal compensation in each phase due to spark gap operation, then the mutual coupling acts to partly correct the imbalance in currents. If the imbalance is due to a single phase fault on a lightly loaded line then the mutual coupling acts to increase the current in the sound phases. However, if a fault is double end fed the mutual effects due to currents fed into the fault from different directions tend to oppose each other and consequently the sound phases are less affected.

Intercircuit mutual coupling produces different effects depending on the loads being carried on the separate circuits. If two circuits feed the same balanced load, as in the study, but one phase of circuit 1 carries reduced current due to a series capacitor being disconnected, then three balanced currents in circuit 2 have no effect on the currents in 1 (providing mutual coupling coefficients between separate phases of the different circuits are the same). However, the currents in 1 act to decrease each of the currents in 2 but do not affect the current balance in this circuit, again provided that mutual coupling does not favour particular phases. Finally, if circuit 1 is faulted and the fault is also fed by circuit 2 via the receiving end bus-bars then the intercircuit mutual coupling acts to decrease the total fault current.

The effect of source impedance in a double circuit system is similar to the effect of mutual impedance,in that currents flowing in one phase of circuit 1 cause a quadrature emf which affects the current in the corresponding phase in circuit 2. Source impedance acts to limit fault current but it also reduces the effect on the transient stability, of disconnecting one circuit. This is because the power transmission capability is not reduced by as much as the proportion of circuits removed.

3.2.2.3 Effect of Simple and Extinguishing Gaps

In a well designed power system, stability should be maintained even if a transmission circuit is disconnected, because line outages usually result from the occurrence of faults. What is important, as far as the spark gap protecting the series capacitor is concerned, is that it should be of the type which least hinders attempts by the system to maintain stability when an external fault has occurred.

Ideally the spark gaps should not operate for external faults, but if this cannot be arranged (by suitably locating the series capacitor or by installing capacitors able to withstand the expected voltage) then it is thought that the extinguishing gap is preferable to the simple gap. This is because affected capacitors will be returned to circuit quickly after the faulted circuit has been removed.

A number of computer programs were run with different parameters and the results indicated that the above statement is correct. The swing curves (the phase separation of the two machines) shown in Fig. 3.7 represent some of the results obtained. The circuit parameters shown in Table 3.1 apply to each of the swing curves. The only parameter changes are spark gap threshold voltage and spark gap type (simple or extinguishing). It was decided to reinstate capacitors protected by a simple gap, after a fixed time delay for illustrative purposes. The acting times for each of the spark gaps for each of the swing curves are shown in Fig. 3.8.

In the period up to fault, each circuit carries the same current in each case and the mechanical power at each machine is constant. When the fault is applied, the power being despatched by machine 1 is increased and that received by machine 2 decreases. The two machines decelerate accordingly. The action of the simple gap reduces the power which is causing machine 1 to decelerate, and for values of spark gap voltage below 180v, this machine accelerates. The action of the extinguishing gap causes

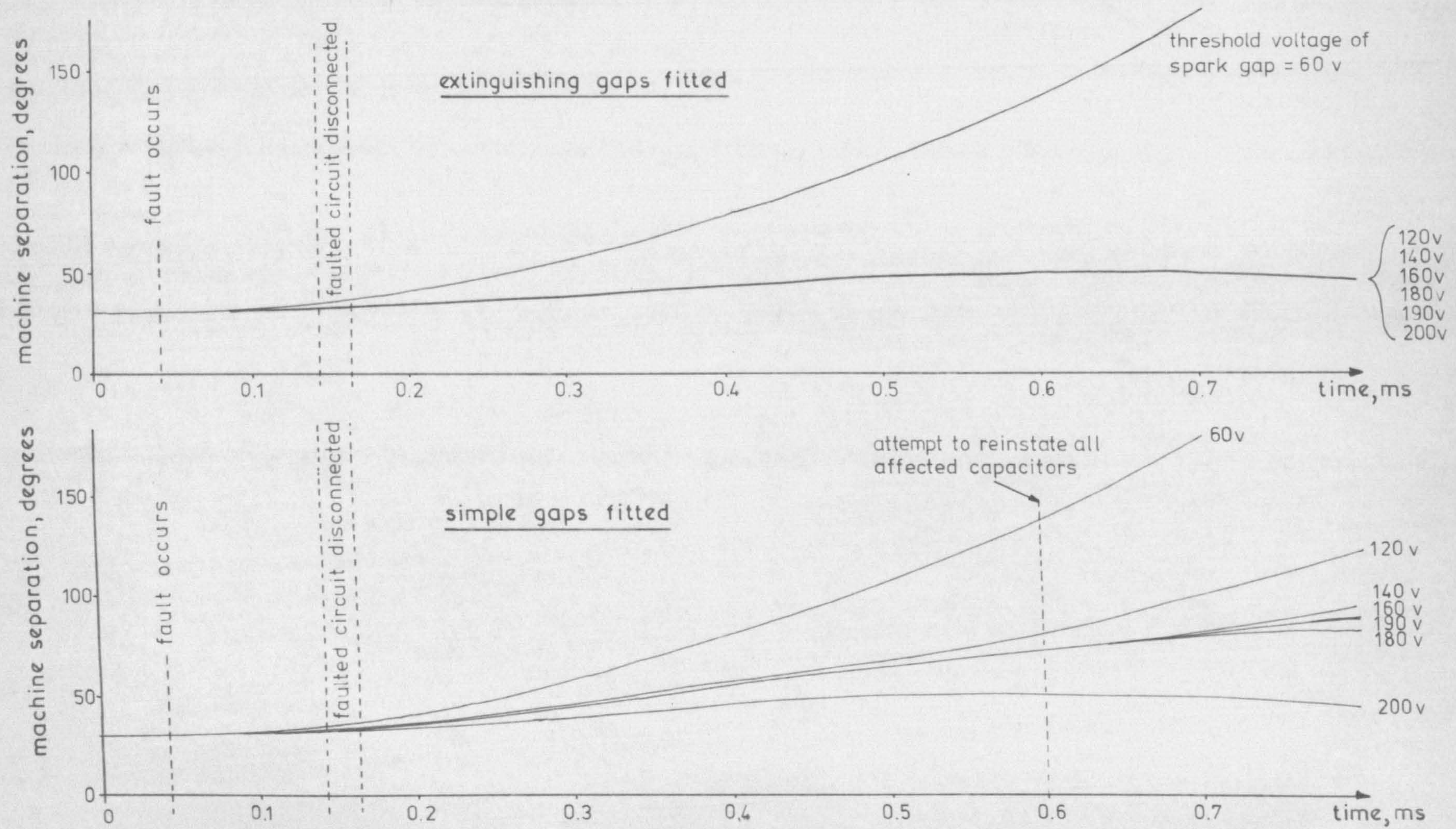


Fig. 3.7 Swing Curves for Different Spark Gaps in Two-Machine System.

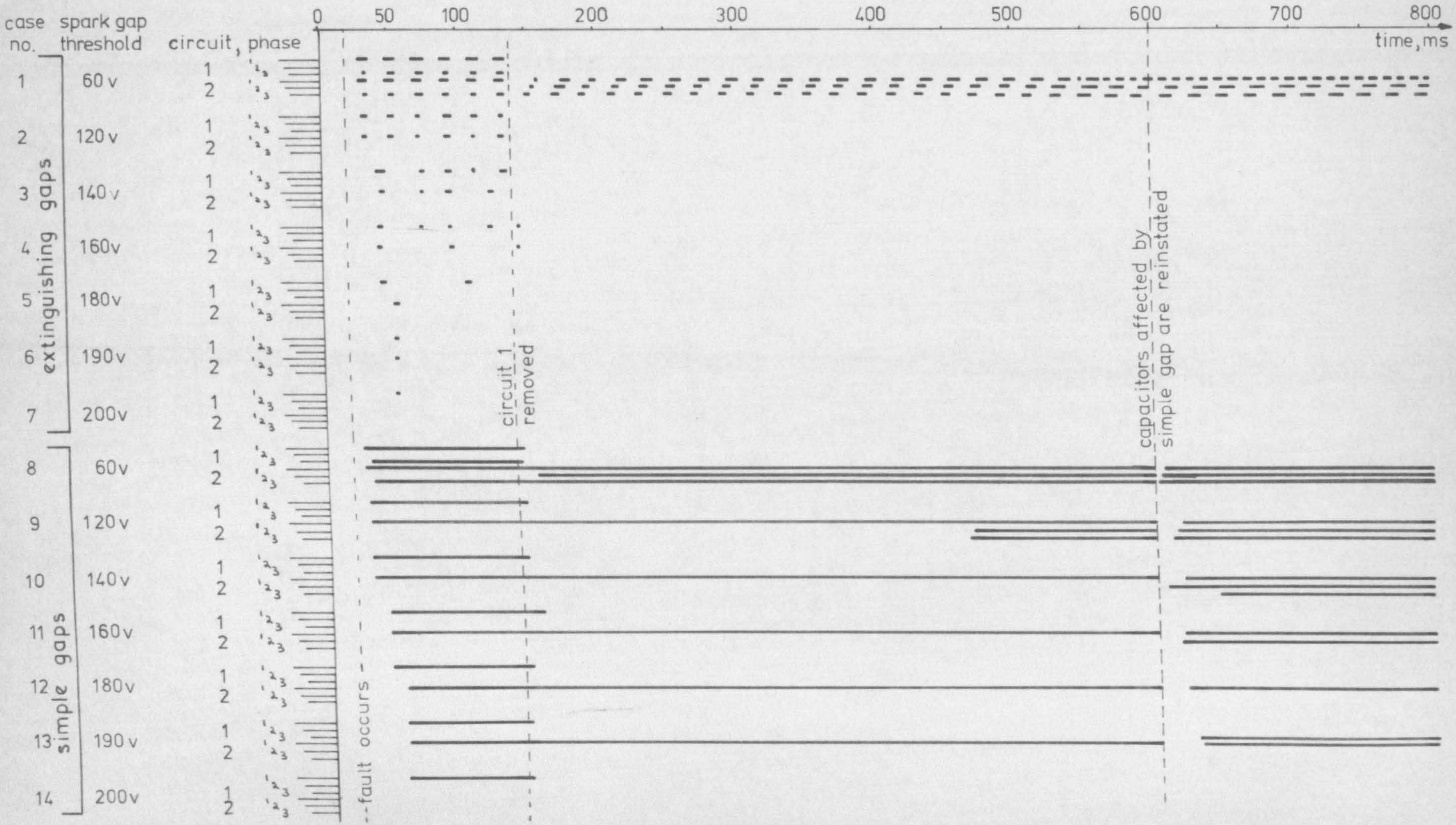


Fig.3.8 Spark Gap Operation with Time (corresponding swing curves - Fig. 3.7)

TABLE 3.1 POWER SYSTEM PARAMETERS USED IN OBTAINING RESULTS IN FIG. 3.7.

$E_{m1} = E_{m2} = 100$ v

$\omega = 100\pi$ rad/sec

$\alpha = 90^{\circ}$

$B_1(0) = 0^{\circ}$

$B_2(0) = -30^{\circ}$

$L_{s1} = 10/\omega$ H

$L_{s2} = 40/\omega$ H

$L = 0.7/\omega$ H/mile

$l = 300$ miles

$x/l = 0.99$

$R = 0.072$ Ω/mile

$M_1 = 0.24/\omega$ H/mile

$R_1 = 0.045$ Ω/mile

$M_2 = R_2 = 0$

Compensation = 75%

$J_1 = 2/9$ joule.sec/degree

$J_2 = 1/9$ " " "

Record starts : t=0 sec, system is in steady-state

L-N fault occurs : t=0.02 sec.

Circuit 1 cleared : between t=0.14 and t=0.16 sec.

Simple gap reinstated at t=0.60 sec.

greater deceleration of machine 1 compared with the equivalent simple gap case and this machine only accelerates during the fault when the spark gap voltage is less than 120v. This is because the capacitor is in circuit for longer, thus permitting greater transfer of power into the fault and also into the spark gap. Machine 2 is less affected by the spark gap type because it is not directly connected with the capacitor in the faulted phase, but this machine decelerates more than machine 1 because not only does it receive less power from machine 1 but it also feeds power towards the fault.

The duration of the fault is short compared with the response times of the machines and the main period which affects the transient stability is, therefore, the post fault period i.e. the period after the faulted circuit has been removed. When extinguishing gaps are fitted, capacitors affected in the healthy circuit are reinstated quickly but when simple gaps are used, this does not happen and the capability of the system to carry stabilizing power between machines is consequently reduced.

In the cases 7 and 14, see Fig. 3.7 and 3.8, the swing curves are virtually identical. This is because the spark gap setting is fairly high and none of the capacitors in circuit 2 operate for the external fault. In cases 1 and 8, the spark gap setting is low and the increased currents which flow during the post fault period, cause persistent extinguishing gap operation in all phases. Clearly this spark gap setting would not be chosen in practice because loads only a little greater than normal would cause spark gap operation. In cases 10, 11 and 13 the premature re-instatement of the simple gap results in spark gap operation on previously unaffected capacitors. This is clearly undesirable and it can be stated that when simple gaps are fitted they should not be re-instated after a fixed time delay but only when the current has reduced to normal load value.

3.3 Transient Analysis of Series Compensated Transmission Lines

A method of solving multiphase power system networks in which mutual coupling between phases and circuits is taken into account, but in which line susceptance is neglected is described in section 3.2. This method is suitable for the stability problem because it is capable of dealing with changing frequency. However, if the supply frequency is constant, then a less complicated method of evaluating circuit current can be used. Accordingly, a computer program, described in section 3.3.1, was written to enable currents to be computed for the single phase fault loop problem.

For the long transmission line problem in which line susceptance is taken into account, waveforms are influenced by travelling waves. A method of dealing with the series compensated transmission line which takes account of travelling waves, is described in section 3.3.2. Results were obtained, for the single phase case, for comparison with the simple fault loop representation.

3.3.1 Transient Analysis of Fault Loop when Line Susceptance is Neglected

The circuit of Fig. 3.9 can represent a fault loop in a power system. The current and voltage at the relaying point are i and v and switch, S , represents the capacitor's protective equipment.

If the generator E.M.F. is known, then the differential equations for the circuit can be readily solved for the two cases a) switch open and b) switch closed. The solutions are, for $e = E\cos(\omega t + \theta)$:-

$$\left.\begin{aligned} i &= e^{-\alpha_o t}(A\cos\beta t + B\sin\beta t) + E\cos(\omega t + \theta - \varphi_0)/Z_0 \\ v &= E\cos(\omega t + \theta) - L_s.pi \\ V_c &= v - Ri - L_L.pi \end{aligned}\right\} \quad 3.16 \text{ (switch open)}$$

$$\left.\begin{aligned} i &= D\,e^{-\alpha_1 t} + E\cos(\omega t + \theta - \varphi_1)/Z_1 \\ v &= E\cos(\omega t + \theta) - L_s.pi \end{aligned}\right\} \quad 3.17 \text{ (switch closed)}$$

where $\alpha_o = R/2L$, $\alpha_1 = R/L$, $L = L_s + L_L$.

$$Z_0^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2, \quad Z_1^2 = R^2 + (\omega L)^2.$$

$$\beta^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2$$

$$\tan\varphi_o = \left(\omega L - \frac{1}{\omega C}\right)/R, \quad \tan\varphi_1 = \omega L/R.$$

A, B and D depend upon initial conditions thus: if S is opened at time $t = t_o$, A and B are determined from:-

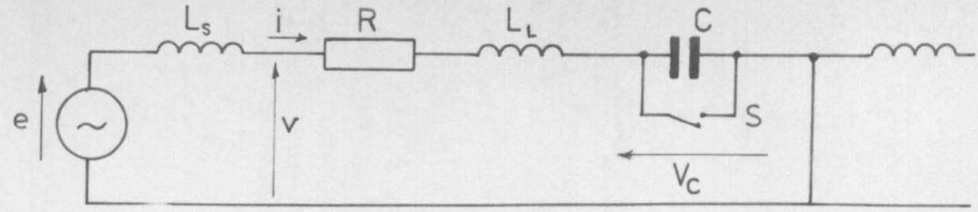


Fig. 3.9 Lumped Parameter Representation of Fault Loop

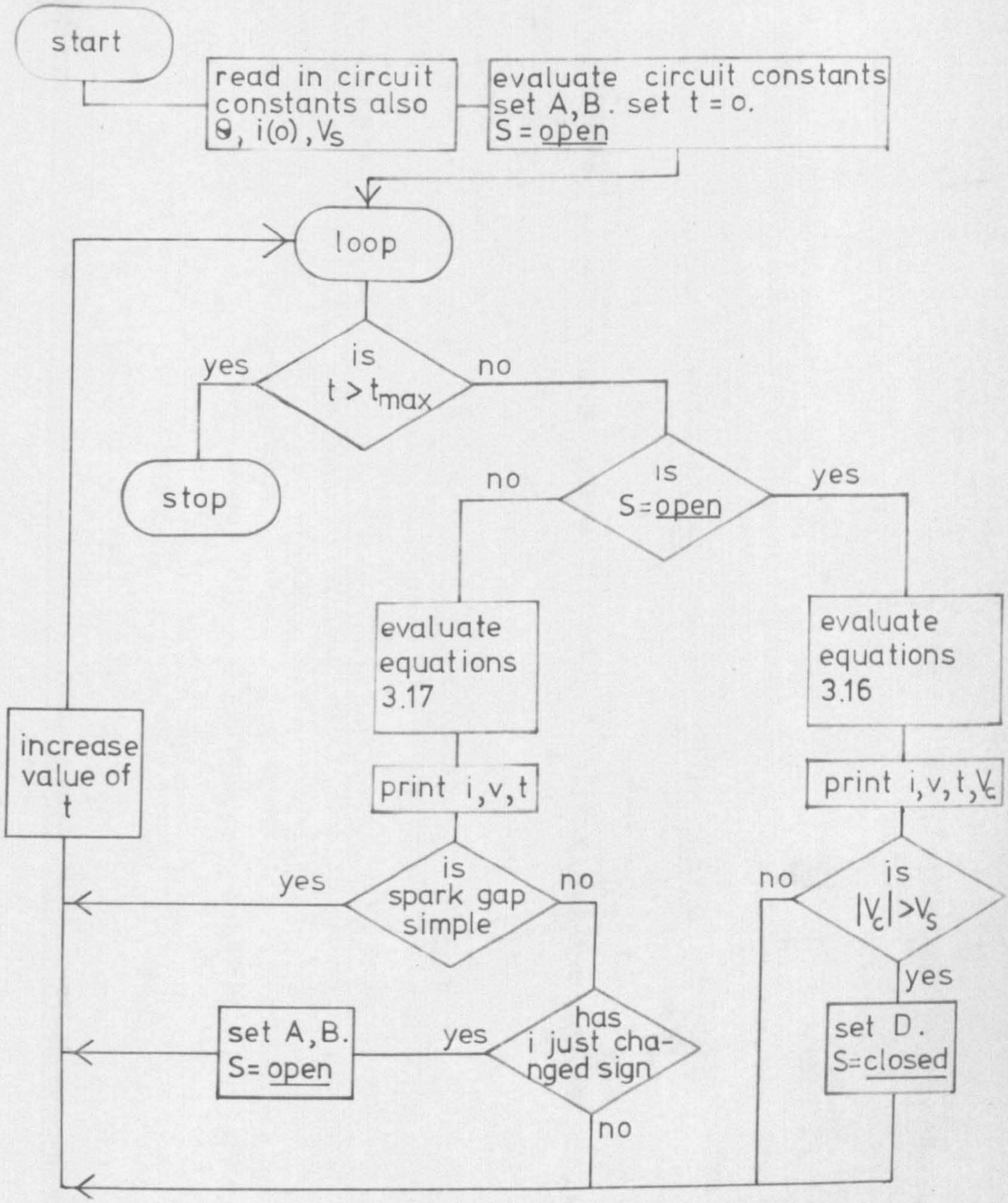


Fig. 3.10 Flow Diagram of Computer Program to Solve Circuit of Fig. 3.9

$$\begin{bmatrix} \cos\beta t_o & \sin\beta t_o \\ -\alpha_o\cos\beta t_o - \beta\sin\beta t_o & \beta\cos\beta t_o - \alpha_o\sin\beta t_o \end{bmatrix} \times \begin{bmatrix} A \\ B \end{bmatrix} = e^{\alpha t_o} \cdot \begin{bmatrix} 1 & -\cos(\omega t_o + \theta - \varphi_o) \\ p & \omega\sin(\omega t_o + \theta - \varphi_o) \end{bmatrix} \times \begin{bmatrix} i(t_o) \\ E/Z_o \end{bmatrix}$$

when S is closed at $t = t_1$, D is given by :

$$D = (i(t_1) - E\cos(\omega t_1 + \theta - \varphi_1)/Z_1).e^{\alpha_o t_1}$$

The values of $i(t_0)$ and $i(t_1)$ in these equations are known because in a circuit in which series inductance is present, the current cannot change instantaneously. The value of $pi(t_0)$ also remains unchanged because at the instant of opening S, the capacitor holds no charge i.e.

$V_c(t_0) = 0.$

Now, if the periods during which S is closed are known,then the complete wave-forms for i and v can be calculated. Since the switch represents a spark gap, the period during which S is closed starts when the capacitor voltage exceeds the setting of the gap i.e. $|V_c| > V_s$. If S represents a magnetic type extinguishing gap, the switch closed period ends when the current next passes through zero i.e. when the sign of i changes.

It proved expedient to program a digital computer to calculate the wave-forms for i and v. The Flow Diagram for the program, Fig. 3.10, shows how the equations were organised for presentation to the computer. The diagram assumes that the spark gap is initially extinguished and the computed results give values of i, v, V_c for different values of t, $0 < t < t_{max}$.

$$\begin{bmatrix} \cos\beta t_o & \sin\beta t_o \\ -\alpha_o\cos\beta t_o - \beta\sin\beta t_o & \beta\cos\beta t_o - \alpha_o\sin\beta t_o \end{bmatrix} \times \begin{bmatrix} A \\ B \end{bmatrix} = e^{\alpha t_o} \cdot \begin{bmatrix} 1 & -\cos(\omega t_o+\theta-\varphi_o) \\ p & \omega\sin(\omega t_o+\theta-\varphi_o) \end{bmatrix} \times \begin{bmatrix} i(t_o) \\ E/Z_o \end{bmatrix}$$

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3.3.2. Transient Analysis of Power System Networks Including the Effect of Distributed Parameter Components

The method described[31] was used to evaluate the currents and voltages in a power system which contained distributed parameter components, namely transmission lines, as well as the lumped parameter components inductance, capacitance and resistance.

Initially loss-less transmission lines were considered since it has been shown that by including lumped resistors at the loss-less transmission line centre and terminals, an attenuation of current and voltage waves is achieved which closely matches the attenuation produced by a distributed resistance in the line.

The equations for a system of transmission lines and lumped parameters are formulated below. Initially the two-port equations are derived for each component in the system, transmission lines, inductors, capacitors etc. and these two-port equations are then incorporated into the nodal equations of the system in order to formulate the appropriate equations for the system as a whole.

3.3.2.1 Loss-less Transmission Lines

The current and voltage at distance x along the line are given by the familiar equations,

$$-\frac{\partial e}{\partial x} = L_1 \frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = C_1 \frac{\partial e}{\partial t}$$

which have solutions, first given by D'Alembert,

$$\left.\begin{aligned} i(x,t) &= f_1(x - Vt) + f_2(x + Vt) \\ e(x,t) &= Z.f_1(x - Vt) - Z.f_2(x + Vt) \end{aligned}\right\} \quad .. \quad .. \quad .. \quad .. \quad 3.18$$

$$\text{where } Z = \sqrt{\frac{L_1}{C_1}}$$

$$V = \sqrt{\frac{1}{L_1 C_1}}$$

The physical interpretation of $f_1(x - Vt)$ is a wave travelling at velocity V in a forward direction and $f_2(x + Vt)$ is a wave travelling in a backward direction.

From equations 3.18 we get:-

$$\left.\begin{aligned} e(x,t) + Z.i(x,t) &= 2\ Z.f_1(x - Vt) \\ e(x,t) - Z.i(x,t) &= -2\ Z.f_2(x + Vt) \end{aligned}\right\} \quad .. \quad .. \quad .. \quad .. \quad 3.19$$

If an observer travels along the line in a forward direction at velocity V then $(x - Vt)$ is constant and therefore as a consequence of equation 3.19:

$$e(x,t) + Z.i(x,t) \quad = \quad \text{constant.}$$

The observer will travel the entire line in a time $\tau = \frac{d}{V}$ where d is the line length. Using the notation of Fig. 3.11(a) equation 3.20 must therefore hold.

$$e_1(t-\tau) + Z.i_{12}(t-\tau) \quad = \quad e_2(t) + Z.(-\ i_{21}(t)) \quad .. \quad .. \qquad 3.20$$

$$\left.\begin{aligned} \text{rearranging we get } i_{21}(t) &= \frac{1}{Z}.e_2(t) + I_2(t-\tau) \\ \text{and } i_{12}(t) &= \frac{1}{Z}.e_1(t) + I_1(t-\tau) \\ \text{where } I_2(t-\tau) &= -\ \frac{1}{Z}.e_1(t-\tau) - i_{12}(t-\tau) \\ \text{and } I_1(t-\tau) &= -\ \frac{1}{Z}.e_2(t-\tau) - i_{21}(t-\tau) \end{aligned}\right\} \quad 3.21$$

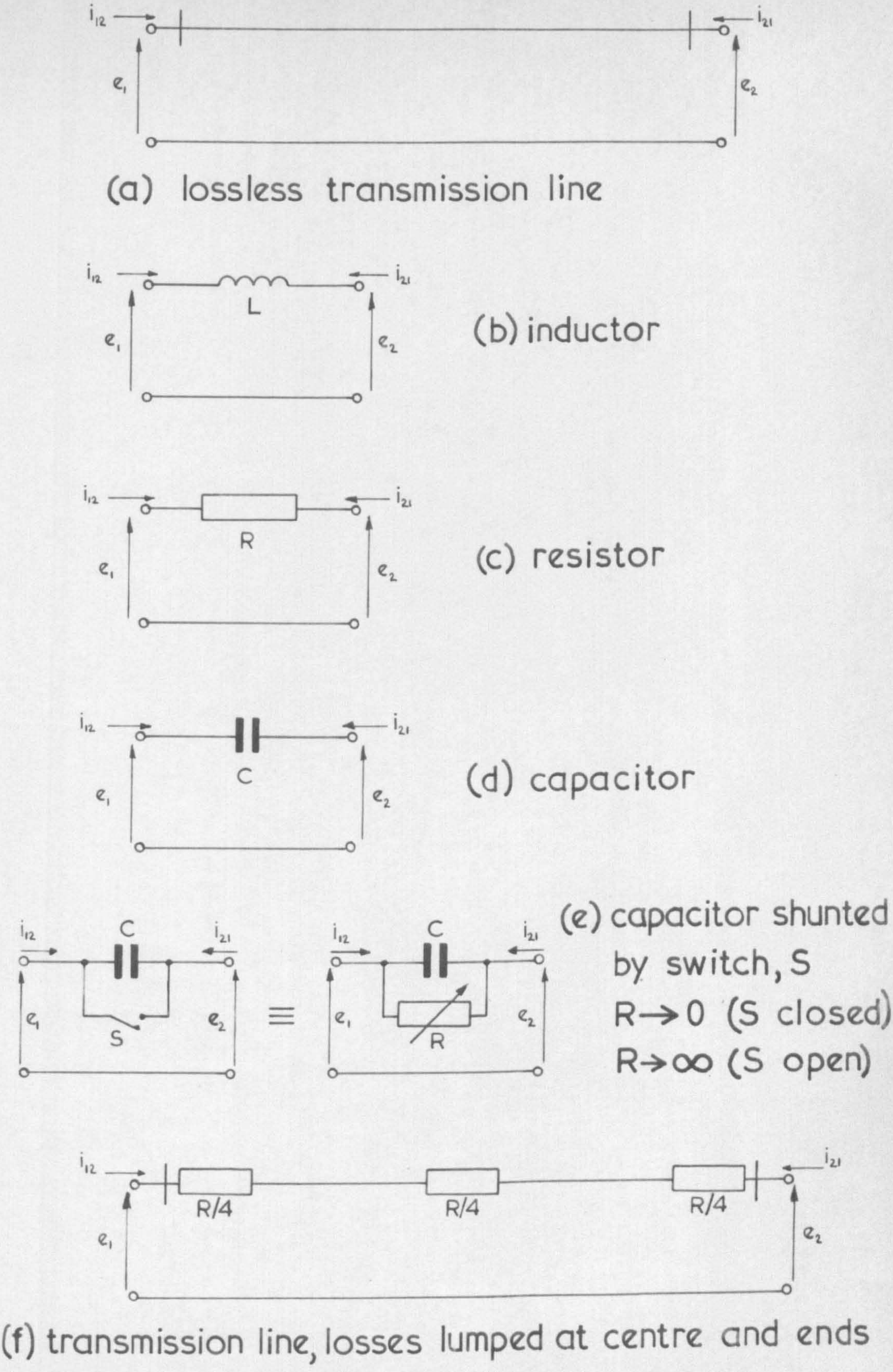


Fig.3.11 Two-port Representation of Circuit Elements.

These are the two-port equations for a loss-less transmission line.

3.3.2.2. Inductors

The circuit of Fig. 3.11(b) is considered.

$$e_2 - e_1 = L.pi_{21}$$

therefore $i_{21}(t) = i_{21}(t-\Delta t) + \frac{1}{L} . \int_{t-\Delta t}^{t} (e_2 - e_1)\, dt$

if Δt is made sufficiently small the integral can be evaluated with little error by using the trapezoidal rule which introduces a truncation error of order $(\Delta t)^3$

hence $i_{21}(t) = \frac{\Delta t}{2L} \cdot (e_2(t) - e_1(t)) + I_{21}(t - \Delta t)$

where $I_{21}(t - \Delta t) = i_{21}(t - \Delta t) + \frac{\Delta t}{2L} .(e_2(t - \Delta t) - e_1(t - \Delta t))$.

3.3.2.3 Resistors

The equation for the two-port network as shown in Fig. 3.11(c) is simply

$$i_{21}(t) = \frac{1}{R} .(e_2(t) - e_1(t)).$$

3.3.2.4 Capacitors

Referring to Fig. 3.11(d).

$$e_2(t) - e_1(t) = \frac{1}{C} . \int_{t-\Delta t}^{t} i_{21}(t)\, dt + e_2(t - \Delta t) - e_1(t - \Delta t)$$

using the trapezoidal rule for the integral yields

$$i_{21}(t) = \frac{2C}{\Delta t} (e_2(t) - e_1(t)) + I_{21}(t - \Delta t)$$

& $I_{21}(t - \Delta t) = -i_{21}(t - \Delta t) - \frac{2C}{\Delta t} (e_2(t - \Delta t) - e_1(t - \Delta t))$.

3.3.2.5 Parallel R - C Combination

When dealing with the parallel combination of a capacitor and a spark gap it is convenient to represent the spark gap by a switch which is either open or closed. The switch can in turn be represented by a variable resistor, R, as shown in Fig. 3.11(e). Since a resistor cannot store energy, suddenly changing the value of R from a very small value to a very large value or vice-versa, cannot suddenly affect the energy content of the circuit of which it forms part.

for the circuit of Fig. 3.11(e)

$$e_1(t) - e_2(t) = \frac{1}{C} \cdot \int_{t-\Delta t}^{t} i_{12}(t) - \frac{[e_1(t) - e_2(t)]}{R} dt + e_1(t-\Delta t) - e_2(t-\Delta t)$$

and using the trapezoidal rule, the two-port equations are obtained:

$$i_{12}(t) = \left(\frac{2C}{\Delta t} + \frac{1}{R}\right) \cdot (e_1(t) - e_2(t)) + I_1(t - \Delta t)$$

$$I_1(t - \Delta t) = - i_{12}(t - \Delta t) - \left(\frac{2C}{\Delta t} - \frac{1}{R}\right) \cdot (e_1(t - \Delta t) - e_2(t - \Delta t))$$

3.3.2.6 Transmission Lines with Series Resistance Approximation

By including half of the series line resistance at the line centre and a quarter of the series line resistance at each of the line terminals as shown in Fig. 3.11(f), it has been found[31] that the results obtained are practically identical to results obtained from classical transient transmission line analysis, at least for the line lengths encountered in power systems.

The appropriate two-port equations, which are developed in Appendix 3.1, are:

$$i_{12}(t) = \frac{1}{Z}\cdot e_1(t) + I_1(t - \tau)$$

$$i_{21}(t) = \frac{1}{Z}\cdot e_2(t) + I_2(t - \tau)$$

$$I_1(t - \tau) = \left(\frac{1 + h}{2}\right).\left(- \frac{1}{Z} e_2(t - \tau) - i_{21}(t - \tau).h\right)$$
$$+ \left(\frac{1 - h}{2}\right).\left(-\frac{1}{Z} e_1(t - \tau) - i_{12}(t - \tau).h\right)$$

$$\& \; I_2(t - \tau) = \left(\frac{1 + h}{2}\right).\left(- \frac{1}{Z} e_1(t - \tau) - i_{12}(t - \tau).h\right)$$
$$+ \left(\frac{1 - h}{2}\right).\left(- \frac{1}{Z} e_2(t - \tau) - i_{21}(t - \tau).h\right)$$

$$\text{where } Z = \sqrt{\frac{L_1}{C_1}} + \frac{R}{4}$$

$$Z.h = \sqrt{\frac{L_1}{C_1}} - \frac{R}{4}$$

3.3.2.7 Analysis of a Complete Circuit

The equations for the circuit of Fig. 3.12 can be readily formulated from the two-port equations described above. In general D_1 and D_2 will be unequal and so travel times of τ_1 and τ_2 for the two transmission lines will also be unequal.

The equations pertaining are:

$$i_{12}(t) = \frac{\Delta t}{2L_{A1}} \cdot (e_1(t) - e_2(t)) + I_{12}(t - \Delta t) \quad .. \quad 3.22a$$

$$i_{21}(t) = - i_{12}(t) \quad .. \quad .. \quad .. \quad 3.22b$$

$$i_{23}(t) = \frac{1}{Z_1}.e_2(t) + I_2(t - \tau_1) \quad .. \quad .. \quad .. \quad 3.22c$$

$$i_{32}(t) = \frac{1}{Z_1}.e_3(t) + I_3(t - \tau_1) \quad .. \quad .. \quad .. \quad 3.22e$$

$$i_{34}(t) = \left(\frac{1}{R_c} + \frac{2C}{\Delta t}\right)(e_3(t) - e_4(t)) + I_{34}(t - \Delta t) \quad .. \quad 3.22f$$

System Diagram

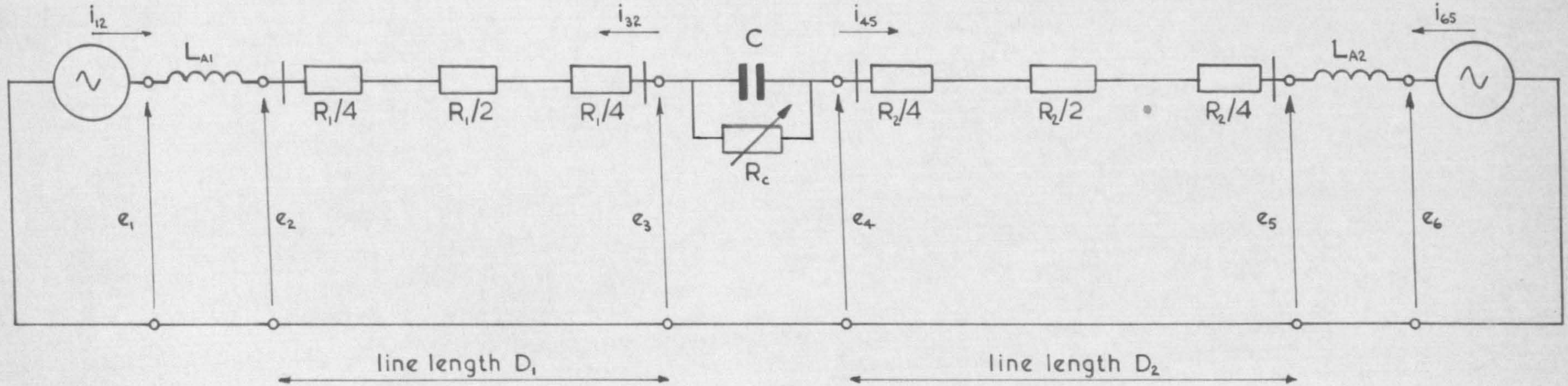


Fig. 3.12 Single Phase Representation of Series Compensated Line with Series Capacitor Located Part-way Along Line

$$i_{43}(t) = - i_{34}(t) \qquad \text{.. .. ..} \qquad 3.22g$$

$$i_{45}(t) = \frac{1}{Z_2} \cdot e_4(t) + I_4(t - \tau_2) \qquad \text{.. .. .. ..} \qquad 3.22h$$

$$i_{54}(t) = \frac{1}{Z_2} \cdot e_5(t) + I_5(t - \tau_2) \qquad \text{.. .. .. ..} \qquad 3.22i$$

$$i_{56}(t) = - i_{65}(t) \qquad \text{.. .. ..} \qquad 3.22j$$

$$i_{65}(t) = \frac{\Delta t}{2L_{A2}} \cdot (e_6(t) - e_5(t)) + I_{65}(t - \Delta t) \qquad \text{.. ..} \qquad 3.22k$$

$$\text{where } Z_1 = \sqrt{\frac{L_1}{C_1}} + \frac{R_1}{4}$$

$$Z_2 = \sqrt{\frac{L_1}{C_1}} + \frac{R_2}{4}$$

The equations for I_{12}, I_2, I_3, I_{34}, I_4, I_5 and I_{56} are needed, and these can be written down easily since it is only necessary to change the subscripts in the formulae presented earlier in the text.

The equations 3.22 can be rearranged and some of the variables eliminated in order to produce equations for the system which can then be solved.

We get:

$$\begin{bmatrix} Q_1 & 1 \\ 0 & 1+Q_1Y_1 \end{bmatrix} \times \begin{bmatrix} i_{12}(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_1(t) \end{bmatrix} + Q_1 \begin{bmatrix} I_{12}(t-\Delta t) \\ I_{12}(t-\Delta t)-I_2(t-\tau_1) \end{bmatrix} \quad ..3.23$$

$$\begin{bmatrix} P_3+Y_1 & -P_3 \\ -P_3 & P_3+Y_2 \end{bmatrix} \times \begin{bmatrix} e_3(t) \\ e_4(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -I_{34}(t-\Delta t)-I_3(t-\tau_1) \\ I_{34}(t-\Delta t)-I_4(t-\tau_2) \end{bmatrix} \quad ...3.24$$

$$\begin{bmatrix} 1+Q_2Y_2 & 0 \\ 1 & Q_2 \end{bmatrix} \times \begin{bmatrix} e_5(t) \\ i_{65}(t) \end{bmatrix} = \begin{bmatrix} e_6(t) \\ e_6(t) \end{bmatrix} + Q_2 \begin{bmatrix} I_{65}(t-\Delta t)-I_5(t-\tau_2) \\ I_{65}(t-\Delta t) \end{bmatrix} \quad ..3.25$$

where $Q_1 = \frac{2L}{\Delta t}A1$, $Q_2 = \frac{2L}{\Delta t}A2$, $P_3 = \frac{2C}{\Delta t} + \frac{1}{R}$

$Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$

In a particular problem e_1 and e_6 are usually known functions of time and i_{12}, e_1, e_3, e_4, e_5 and i_{65} are unknown functions of time.

Each of the equations 3.23 - 3.25 are then of the form

3.26 .... $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$ x and y unknown.

which is readily solved for x and y.

The unknown variables in equations 3.23 - 3.25 can thus be evaluated at time t provided that the terms analogous to f and g in equation 3.26 are known. Thus the time variance of e_1 and e_6 must be known together with some of the history associated with all the unknown variables. If a solution is to be evaluated at time intervals of Δt, it is only necessary to record the most recent values of the variables associated with I_{12}, I_{34} and I_{65} but values over the past periods τ_1 and τ_2 are required for the variables associated with I_2, I_3 and I_4, I_5 respectively.

In order to determine the value of R_c, it is necessary to keep a check on the value of $e_3 - e_4$ and to bear in mind the characteristics of the spark gap which R_c represents. If the spark gap is of the extinguishing type it is also necessary to realise when the sign of $i_3(t)$ does not equal the sign of $i_3(t-\Delta t)$

since this condition may indicate that the spark is extinct and R_c must be made large in consequence.

Since the task of determining a solution to the circuit is to be undertaken on a digital computer, the value of R_c must be finite and should not be made identically equal to zero. Values of $R_c = 10^6$ and $R_c = 10^{-6}$ were found to be representative of open and short circuit conditions when the range of circuit parameters under consideration corresponded to the values found in power systems.

3.3.2.8 Mutual Coupling between Circuits

To include mutual coupling between lumped parameters, the scalar quantities of a single two-port branch are simply replaced by matrix quantities for the set of coupled two-port branches. In the case of loss-less multiphase lines which are mutually coupled the solution is obtained by transforming the actual n phase quantities into n modal quantities. The transformation is arranged in such a way that the differential equations for the line expressed in modal quantities become independent for the n modes. This means that the actual n phase line is transformed into n independent single phase lines in the modal domain. Each mode is defined by a modal travel time and a modal surge impedance. The mathematics of the required transformations is described in reference 32.

3.3.2.9 Frequency Dependent Parameters

The nature of frequency dependent parameters is discussed in chapter 2. The problem of incorporating these parameters into a transient analysis of a power system has recently been solved, both in this country[21] and in America[33] by operating upon the equations in frequency domain. Although frequency dependent parameters can easily be taken into account when operating in frequency domain, certain simple time domain phenomena, such as sequential closing and opening of the three phases of a circuit breaker are very difficult to represent. It may be possible to include the effect of the series capacitor protection but this has not yet been achieved. For this reason and also because of the practical complexity of the approach, frequency dependent parameters have been held constant in the analysis contained in this thesis.

3.3.2.10 Fault Impedance

Fault resistance or any other non-linear resistance can easily be taken into account provided that the resistance is a known function of current and/or voltage or else is known as a function of time. Although a number of arbitrary formulae are in use to represent fault arc resistance, it is felt that a simple square wave-form of arc voltage is adequate for most purposes. In a digital computer calculation the recommended function for arc resistance, R_a, is then:

$$R_a(t) = \frac{E}{|i(t - \Delta t)|} \quad ; \quad |i(t - \Delta t)| \geqslant \varepsilon$$

$$R_a(t) = 0 \qquad ; \quad |i(t - \Delta t)| < \varepsilon$$

where E is the amplitude of the arc voltage and ε is an arbitrarily small positive number.

3.3.3 Use of Transient Analysis Methods

In an earlier work, Bruce[34] obtained waveforms equivalent to the relaying current and voltage in a single phase series compensated fault loop. His wave-forms, recorded in the laboratory from a line model consisting of 30 'T' sections of line and operating at 5 kHz, correspond to line lengths, up to fault, of 230 miles. It has thus been possible to check that the wave-forms predicted by the analysis described above are correct for the particular circuits which have been studied.

In chapter 4 the methods described for analysing power system circuits are used to evaluate the design and effectiveness of transmission line protective relays when applied to series compensated lines.

One of the advantages of using computer produced results is that the wave-forms can be easily scrutinised; the noise and measurement errors associated with experimental work are not present.

Two sets of wave-forms are shown in Fig. 3.13 and Fig. 3.14. They are examples of the type of wave-forms produced on a series compensated transmission line during faults, as calculated by a digital computer using the method described above. Fig. 3.13 shows the relevant currents and voltages

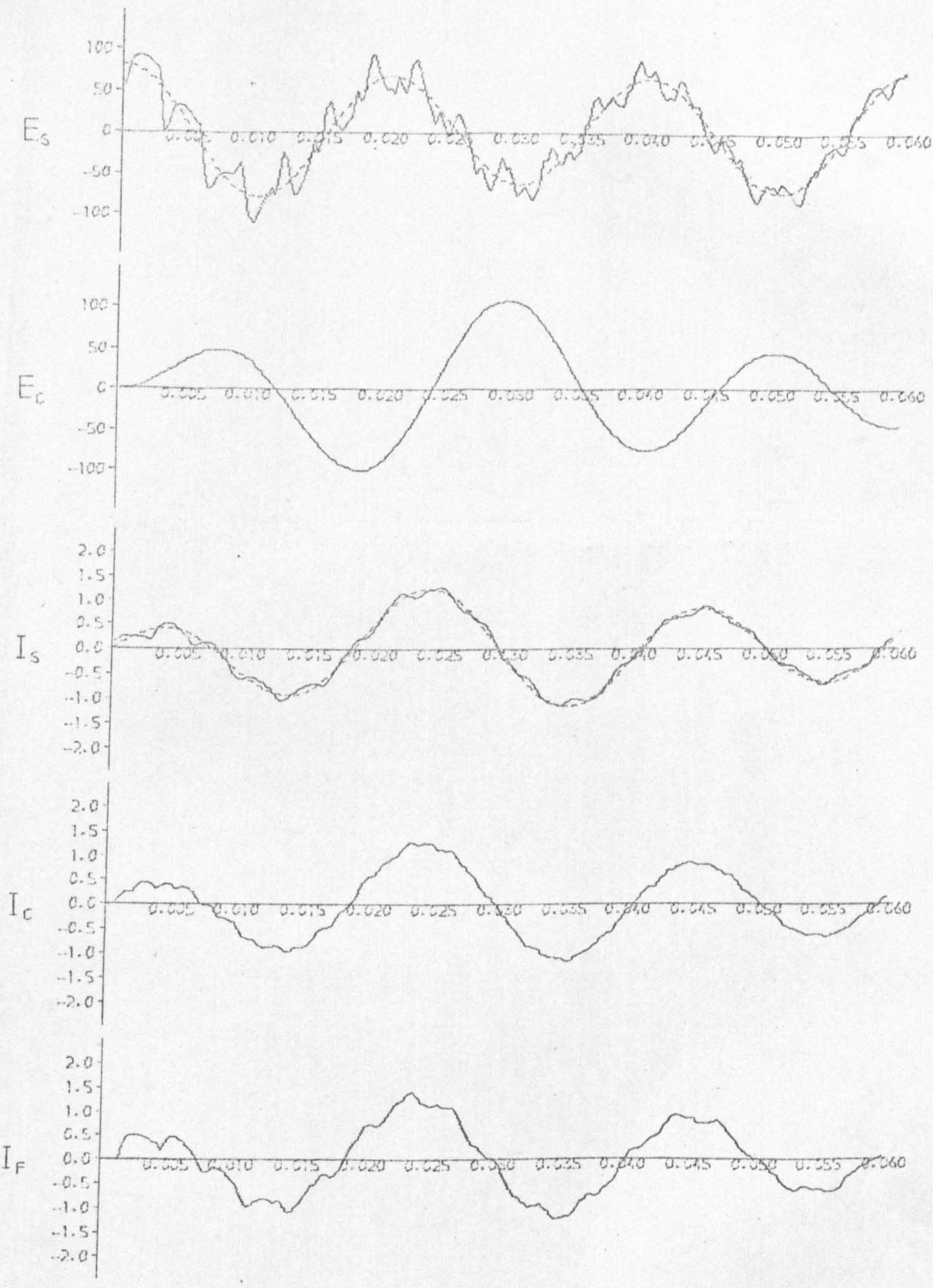


Fig.3.13 Wave-forms in circuit of Fig.3.15. Line energised at voltage maximum. Spark gap does not operate.

------- WAVE-FORMS OBTAINED FOR ZERO LINE SUSCEPTANCE.

——— WAVE-FORMS OBTAINED, LINE SUSCEPTANCE TAKEN INTO ACCOUNT.

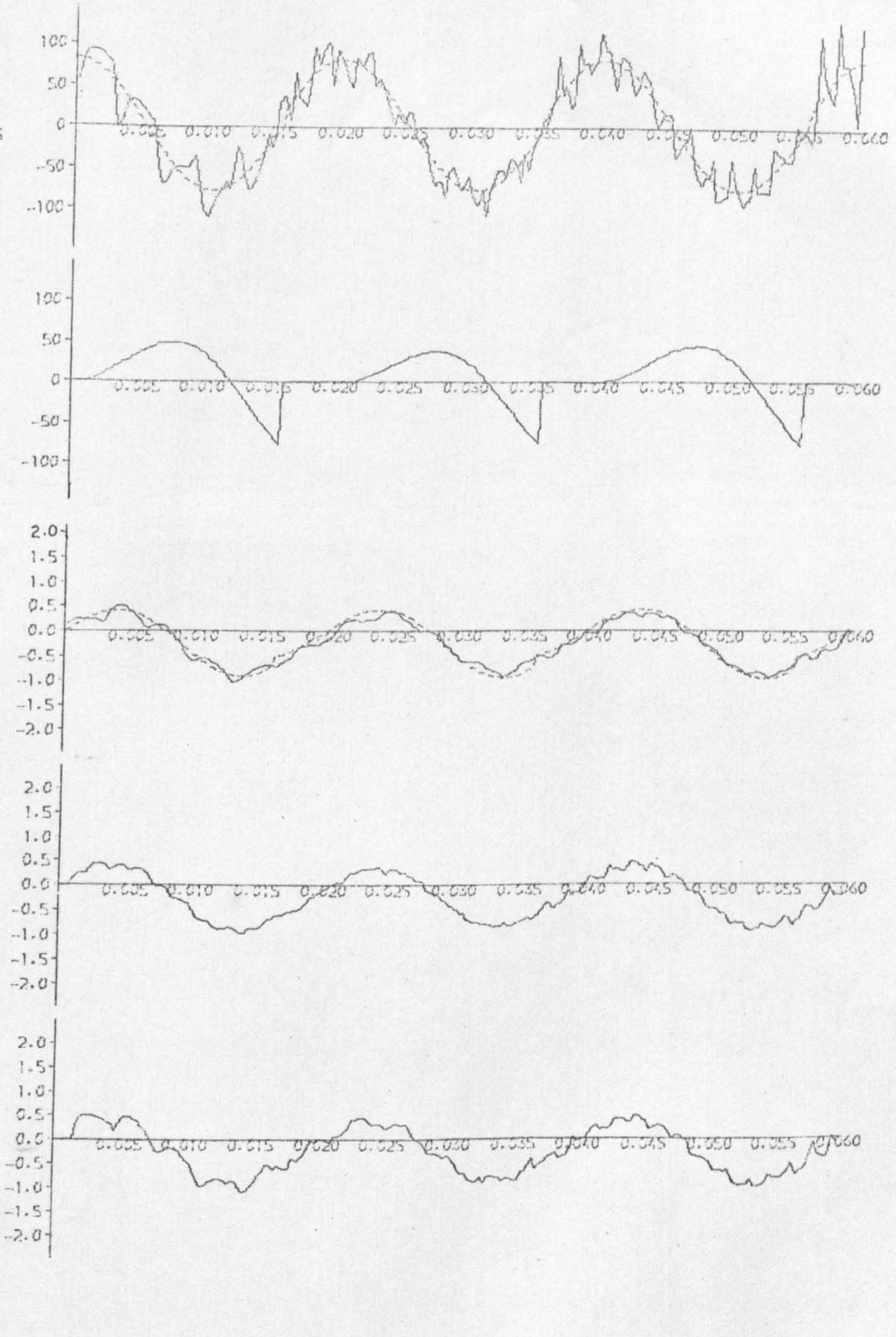


Fig. 3.14 Wave-forms in circuit of Fig.3.15. Line energised at voltage maximum. Extinguishing gap operation occurs

- - - - - - WAVE-FORMS OBTAINED FOR ZERO LINE SUSCEPTANCE.
——— WAVE-FORMS OBTAINED, LINE SUSCEPTANCE TAKEN INTO ACCOUNT.

in the circuit of Fig. 3.15 when the capacitor's protective spark gap does not operate. Fig. 3.14 shows wave-forms corresponding to the same circuit except that the capacitor is protected by an extinguishing gap set at a typical value for the system. The current, I_s and voltage, E_s at the relaying point, have also been calculated on the assumption that line susceptance is negligible and these wave-forms are presented 'broken', for comparison with the calculated values which take account of line susceptance.

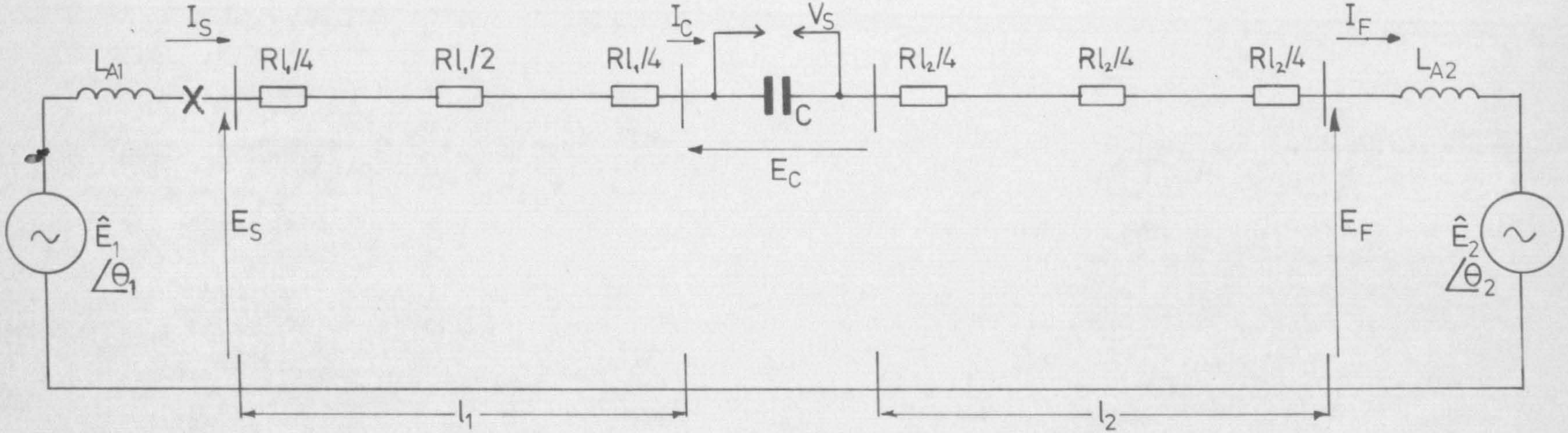
The salient features of the wave-forms are:

1. The effect of the travelling waves is to introduce a significant number of high frequencies into the current and voltage waves.

2. In the circuit in which the spark gap does not operate the wave-forms follow an alternating pattern. In the case of repetitive spark gap operation, a considerable and persistent d.c. offset is introduced into the relaying current. In the case of a purely inductive source impedance this offset does not appear in the relaying voltage.

3. The spark gap operation introduces extra travelling wave-fronts into the system, thus causing an increase in the high frequency distortion of the voltages. The currents are also distorted but the source inductance eliminates the sudden current jumps at times of reflection.

4. All high frequencies are attenuated by the line



ωL_{A1} Ω	ωL_{A2} Ω	R Ω/ml	ωL_1 Ω/ml	ωC_1 μmho	$1/\omega C$ Ω $50\% \times (l_1+l_2)\omega L_1$	$\hat{E}_1$ V	$\hat{E}_2$ V	θ_1	θ_2	V_S V	l_1 ml	l_2 ml	
33.34	0.1	0.08515	0.656	5.87	..	100	1	$\pi/2$	$\pi/6$	1000	115	113	Fig. 3.13
..	..	..	..	..	..	..	..	..	..	80	..	..	Fig. 3.14
..	..	..	..	..	..	..	..	..	..	..	57.5	56.5	Fig. 3.16
0.1	..	..	..	..	..	..	..	..	..	..	115	113	Fig. 4.8
0.1	..	..	..	..	..	..	..	0	..	..	..	..	Fig. 4.9

Fig.3.15 Representation of Single Phase Fault, Distance ($l_1 + l_2$) Along Line

resistance but when fresh discontinuities are introduced each time the capacitor is discharged, the current and voltage wave-forms are never free from high frequency distortion.

The effect of a shorter line is to increase the high frequency distortion in the voltage but to decrease the overall distortion in the current. Fig. 3.16 illustrates this. Clearly, if the travelling wave-fronts take less time to traverse the line then discontinuities are received at the sending end more frequently, and since source impedance is not zero, sudden increases in current are not permitted and, therefore, the sending voltage must accommodate the surplus energy.

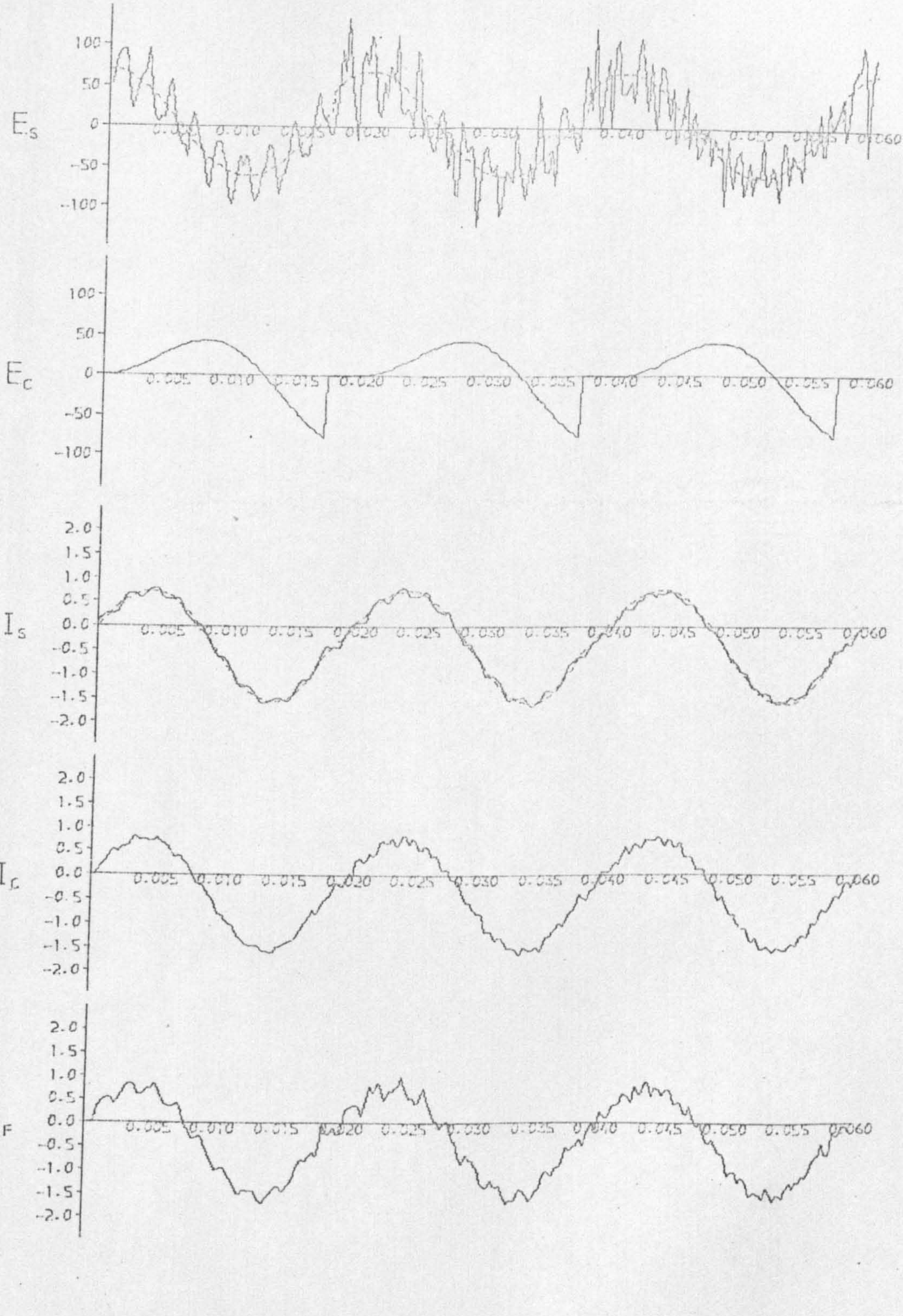


Fig. 3.16 Wave-forms in circuit of Fig. 3.15. Line energised at voltage maximum. Extinguishing gap operation occurs

- - - - - - - WAVE-FORMS OBTAINED FOR ZERO LINE SUSCEPTANCE.

———— WAVE-FORMS OBTAINED, LINE SUSCEPTANCE TAKEN INTO ACCOUNT.

4. LINE PROTECTION PROBLEMS

In this chapter the existing techniques for protecting transmission lines are described together with the special problems associated with series compensated lines. In section 4.2 the idea of basing line protection on travelling waves is critically examined and then methods based upon the ordinary differential equation of a fault loop are considered. One of these methods, based upon sampling of relaying wave-forms, was chosen for investigation with a view to eventually constructing a prototype relay. The investigation is described in section 4.3 and the design and construction of the prototype relay is dealt with in chapter 5.

4.1. Existing Methods

The process of detecting faults should be fast, i.e. should take less than 0.1 seconds, Relays should discriminate between internal line faults and faults on adjacent lines and the equipment must have a high reliability.

The most satisfactory protection arrangement is to compare the currents and/or voltages at one end of a line with the currents and voltages at the other end. Unfortunately, it is very difficult to communicate any detailed information between the ends of a long line, since pilot cables are costly and have technical shortcomings over long distances and radio communication has similar drawbacks.[35] A high frequency carrier signal can be injected along the actual transmission line conductors, but the EHV lines are not designed with such uses in mind and in practice this means that the information sent and received must be of the simple one binary bit form. Evenif a carrier channel is installed, it is not customary to rely absolutely upon it since carrier equipment failures do occur. The most suitable class of protection for long transmission lines is, therefore,Distance Protection.

Distance Protection aims to measure the distance of a fault by using information extracted from the voltages and currents at a line end. In all currently used theory it is assumed that these quantities are in steady-state equilibrium. Thus, it is acceptable to refer to transmission line reactance and impedance. In fact, the time scale usually involved is so short that such equilibrium is never attained.

4.1.1 Basic Distance Protection

Relay operation, see Fig. 4.1 depends upon the assumption that the distance between the relaying point and the fault is proportional to the ratio of the two input quantities i.e. $\frac{V}{I} = Z_F$. The setting of the relay is defined as Z_N, an impedance having the same angle as the transmission line impedance and which just causes the relay to operate for $Z_F = Z_N$.

Protection of a given line with only one relay is not practical because Z_N cannot be set accurately enough to cover the transmission line without falling short, so leaving some

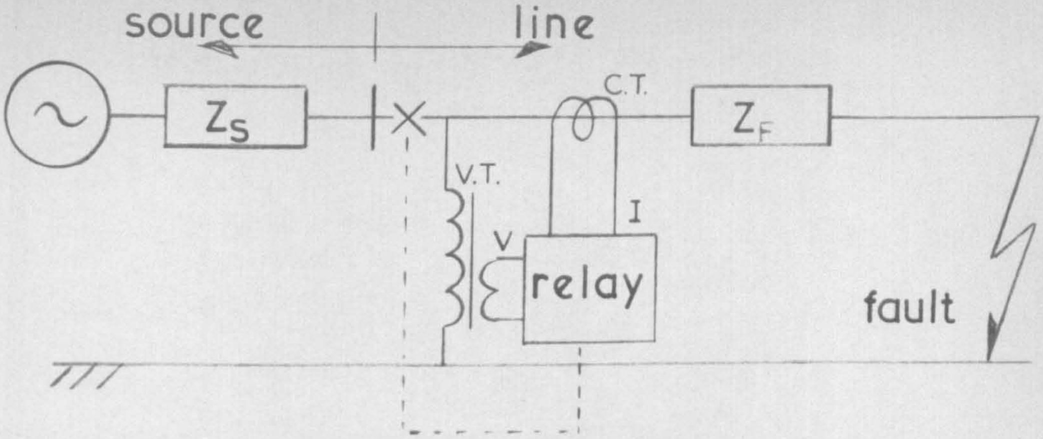


Fig. 4.1 Schematic of Power System

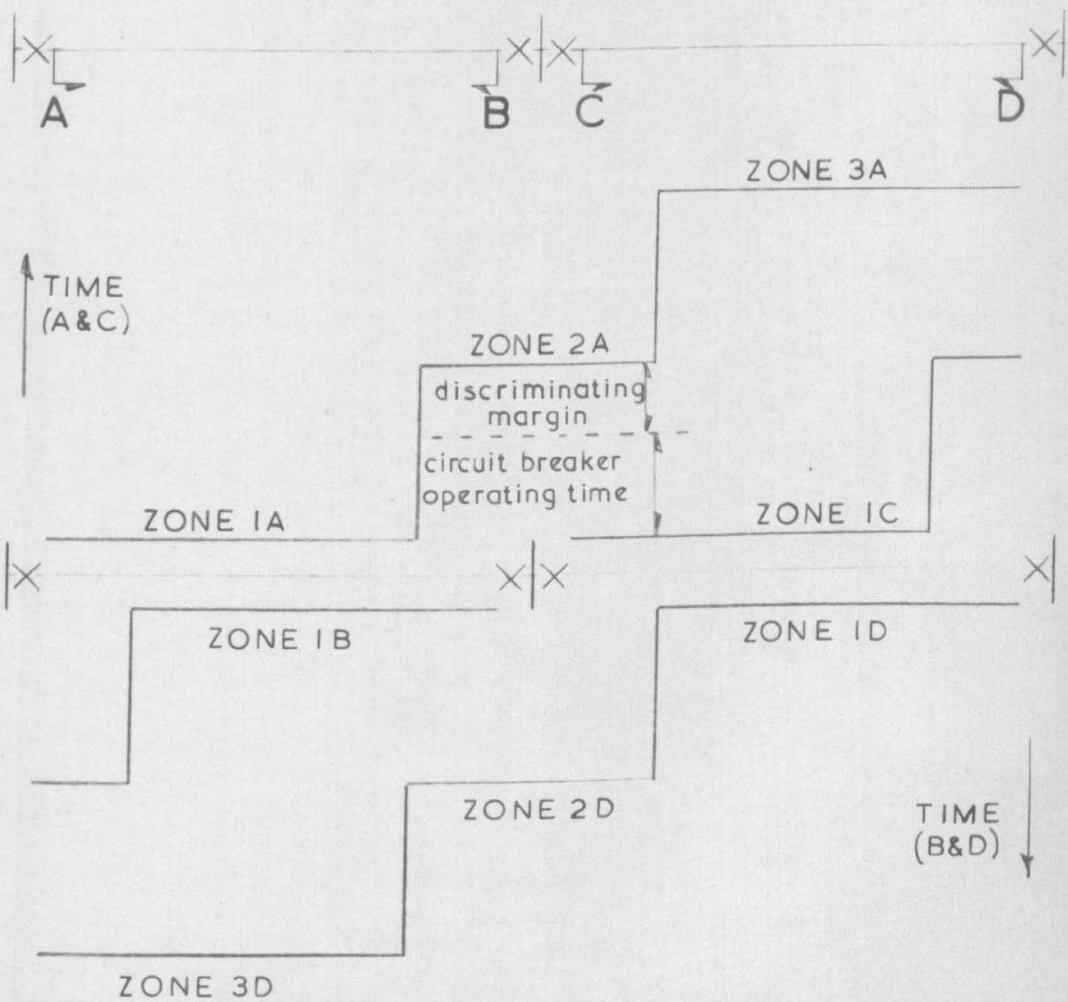


Fig. 4.2 Time-distance Diagram of Distance Protection

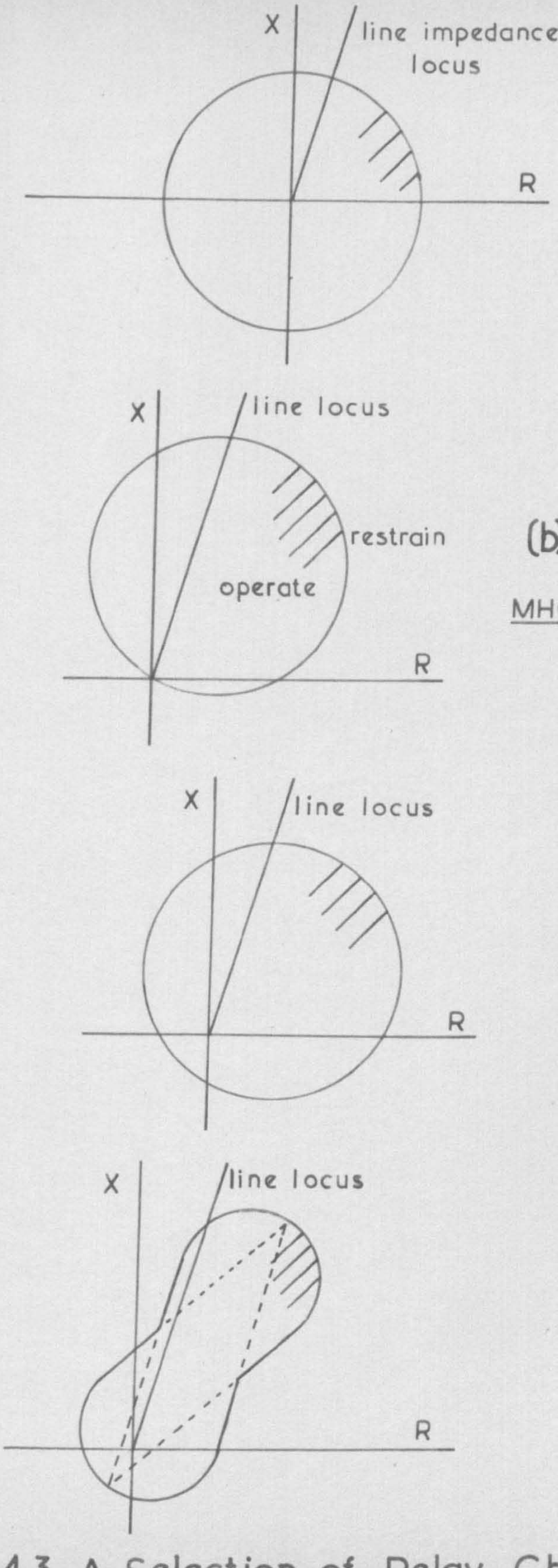
unprotected line, and without overreaching, so allowing relay operation for faults beyond the transmission line being protected. The conventional method of obtaining discrimination is to use a 3 zone arrangement of relays as shown in Fig. 4.2. Settings are chosen so as to cover inaccuracies in relays and assessment of system impedances and may be varied depending upon system configuration. Usual values are

Zone 1 - 80% of line length.

Zone 2 - 120% of line length.

Zone 3 - 200% of line length.

Zone 2 and Zone 3 relays incorporate definite time lags in order to achieve discrimination between relays on adjoining line sections. For complete protection of a 3 phase system separate phase fault and earth fault relays are required, each with definite selection of input quantities. This is necessary to ensure that the relay always responds to the positive sequence impedance of the line. A complete distance protection scheme therefore requires a total of 18 elements all of which continually monitor the system. A less complex arrangement which has often found favour on grounds of size and cost is to use a switched zone 1/2 relay. A further saving in equipment can be obtained by using a switched distance relay scheme, employing only one measuring element. Starter relays of either the overcurrent, undervoltage or impedance type ensure that the single measuring element receives the correct voltages and currents for a particular fault condition. The operating time is greater and reliability may not be as high as the full 18 element scheme. These factors frequently lead to this relay only being used on less important lines or for back-up applications. The impedance up to fault, Z_F, will not necessarily have the same angle as the relay setting, Z_N, and the performance of the relay can therefore only be described adequately by considering a characteristic in the impedance plane. A number of different characteristics which are used are shown in Fig. 4.3. The simple circular impedance characteristic does not distinguish between faults occurring in different directions and is only used in conjunction with other relays. When mho relaying is used to protect any given line section, its operating characteristic encloses the least space in the R - X diagram which means that it is least affected by abnormal system conditions i.e. disturbances other than line faults; in other words, it is the most selective of all simple distance relays. Because the mho relay is affected by fault arc resistance more than other relay types, it is frequently applied to longer lines where it is used for Zone 1 and Zone 2 coverage. The offset mho is frequently used to cover Zone 3 operation. In modern distance relay construction, semiconductors are used extensively and this has permitted the design of non-circular characteristics, see Fig. 4.3(d), more able to resist operation due to load encroachments and more able to identify high resistance faults for example. Each characteristic is based upon the mixing together of the input signals to the relay and the formation



(a) PLAIN IMPEDANCE CHARACTERISTIC

(b) MHO CHARACTERISTIC

(c) OFFSET MHO CHARACTERISTIC

(d) OFFSET SHAPED CHARACTERISTIC

Fig.4.3 A Selection of Relay Characteristics in the Impedance Plain of two signals S_O and S_R which are compared in amplitude or more usually the formation of two signals S_1 and S_2 which are compared in phase. The theory which supports this mixing together of signals is well covered in the references. Certain errors which are associated with the distance relay have received attention in the literature. They are the effect of interphase mutual coupling and inter-circuit mutual coupling,[36,37] the effect of non transposition of line conductors[38,39] and the presence of transients in the relaying waveforms.[40] Distance relays are constructed with a wide range of characteristics as presented in the impedance plane and some compensation can be made for the errors mentioned above.

When choosing a particular relay characteristic, special consideration should be given to three factors.

a) The impedance measured by the relay when load currents are flowing should not lie within the relay operating characteristic. With the very high setting impedances which can be associated with systems of long lines, load encroachment may be possible. Fig. 4.4. shows the normal load regime in relation to a mho characteristic having a fairly high setting. In the case in point it would be necessary to modify the characteristic, possibly in the manner indicated in the figure.

b) The locus of the impedance that would be measured by a relay under power swing conditions is of the form ABCDE in Fig. 4.5 (a). This should not cause a healthy circuit to be

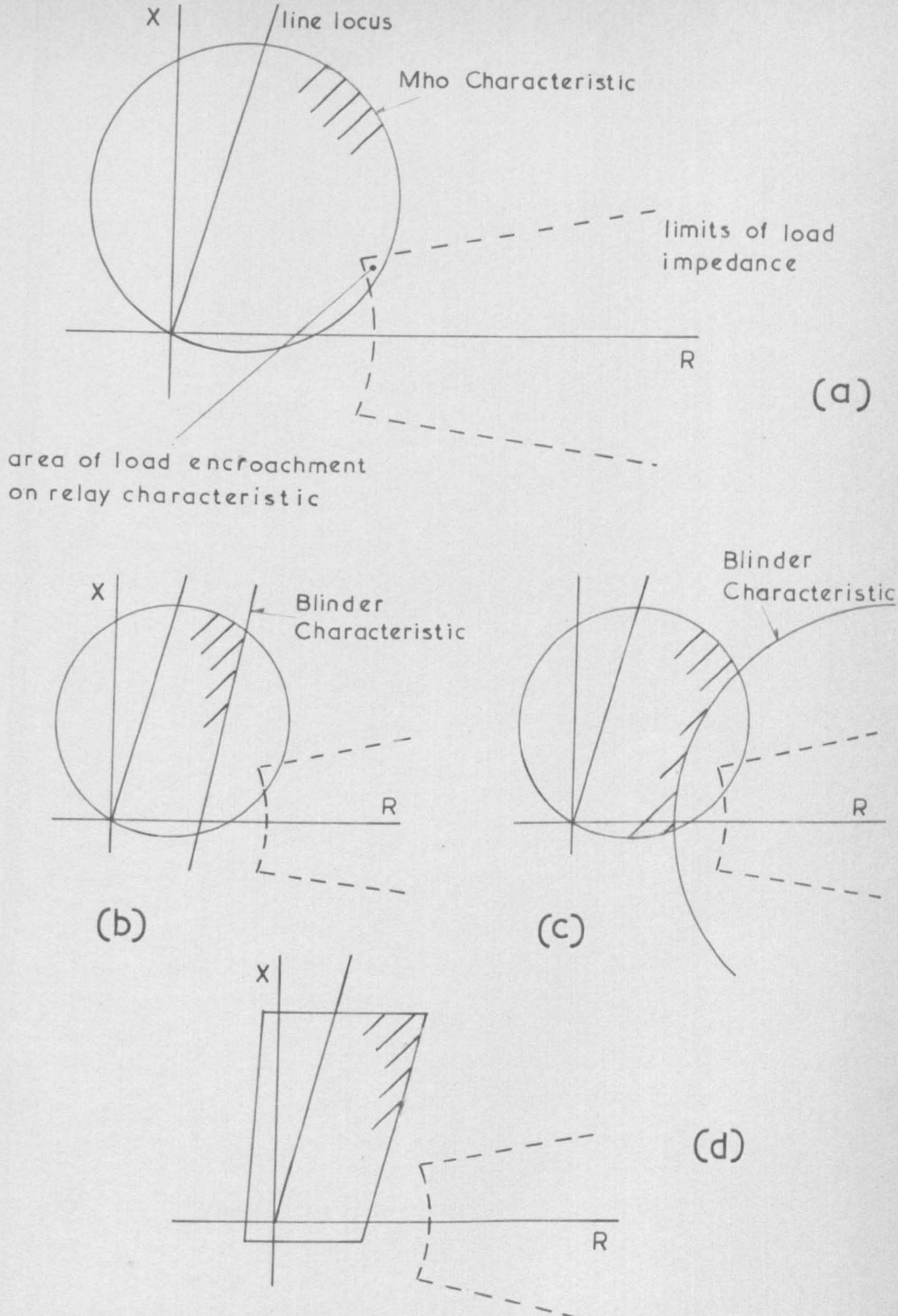
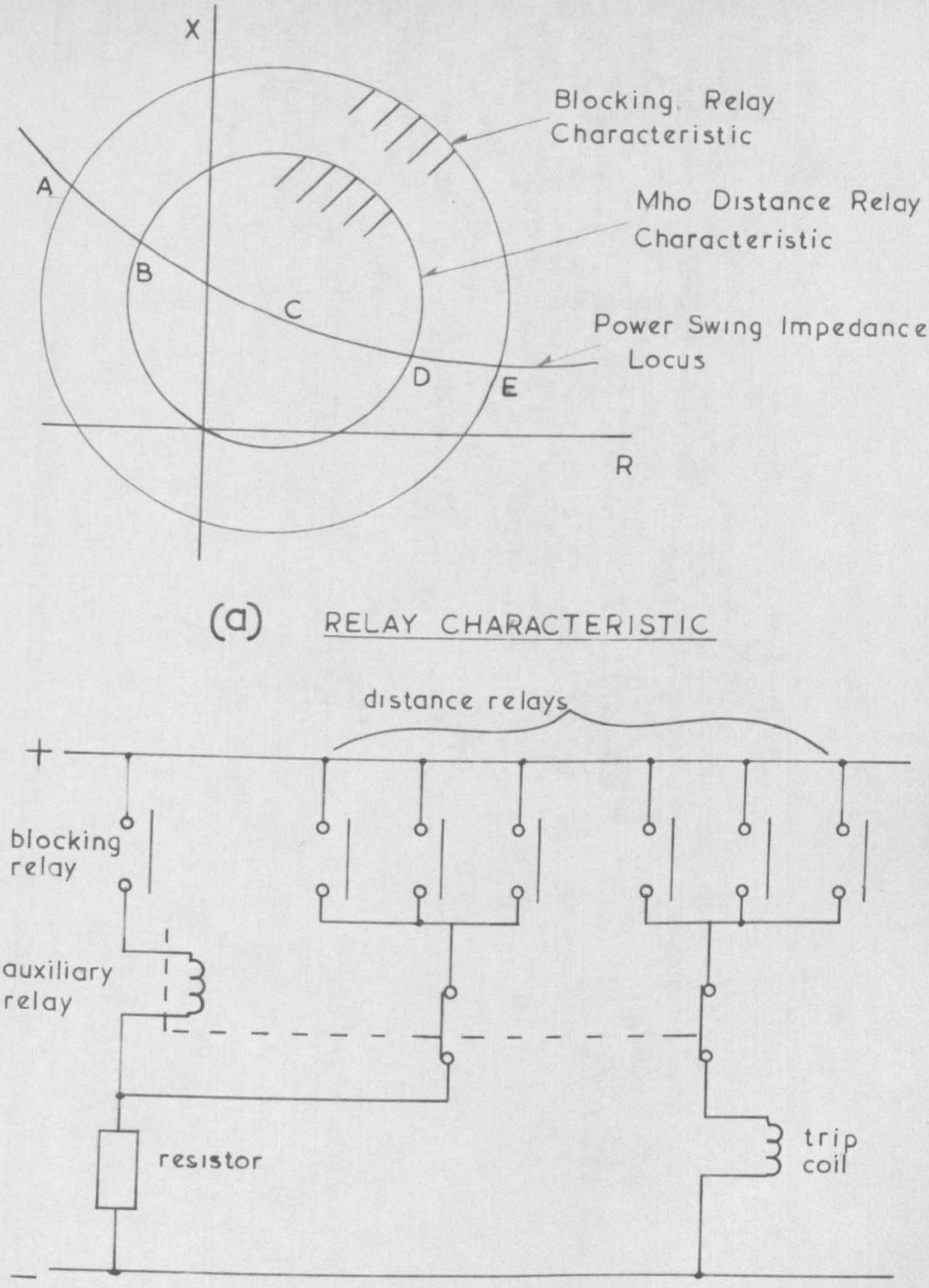


Fig.4.4 Effect of Load Impedance Locus Showing:-
(a) Encroachment on Mho Circle,
(b&c) Solution Using Blinder Characteristics,
(d) Solution Using Quadrilateral.



(a) RELAY CHARACTERISTIC

(b) TRIPPING CONTACT ARRANGEMENT

Fig. 4.5 A Scheme for Trip Blocking Under Power Swing Conditions tripped out. To prevent incorrect tripping a blocking relay having a characteristic which encloses the basic distance relay characteristic is used. As the power swing, which is a balanced three phase phenomenon, causes the apparent impedance to lie between points D and E on the locus the blocking relay operates and causes energisation of an auxiliary relay connected as shown in Fig. 4.5 (b). Subsequent operation of the distance relay would then be ineffective until after the power condition has disappeared. If the point C is an impedance measurement due to a fault,the blocking relay would be unable to cause the relatively slow auxiliary relay to pick up because the distance relays are faster to operate and would succeed in bypassing the auxiliary relay coil.

c) The likelihood of very high resistance faults along a particular transmission line should be considered because this may require special shaping of the distance relay characteristic. The factors which cause resistance faults are :

1) arc resistance - the arcs may be across an insulator to a tower or drawn between phases following whipping or clashing of conductors. Many empirical formulae have been used which take into account wind velocity and arc length but such formulae cannot take account of every possible condition.

2) high ground resistance - occasionally a conductor breaks or sags and touches the ground. The ground contact

resistance of such a fault will be much higher than the tower footing resistance. In such situations, especially in very dry areas (e.g. deserts) very high fault resistance can be present.

3) Vegetation faults - It is desirable to keep the vicinity of a major transmission line clear of vegetation but in certain parts of the world, in tropical areas for instance, it may be less costly to suffer occasional damage and inconvenience due to faults caused by growing vegetation than to keep the entire route clear of such intrusion. In other cases it may not even be possible to eliminate vegetation.

If there are sources of energy at both ends of a line then current will feed into a resistive fault from both ends and this causes measured impedance to lie along loci as shown in Fig. 4.6. Ideally the actual relay characteristic used should envelop these loci. An example is shown in the figure.

4.1.2. Extension of Principles to Series Compensated Lines

A series capacitor can upset the basic premise on which the principle of distance relaying is founded. The basic premise is that the ratio of voltage to current at a relay location is a measure of the distance to a fault. A series capacitor introduces a discontinuity into the ratio of voltage to current and particularly into the reactive component of that ratio as a fault is moved from the relay location toward and

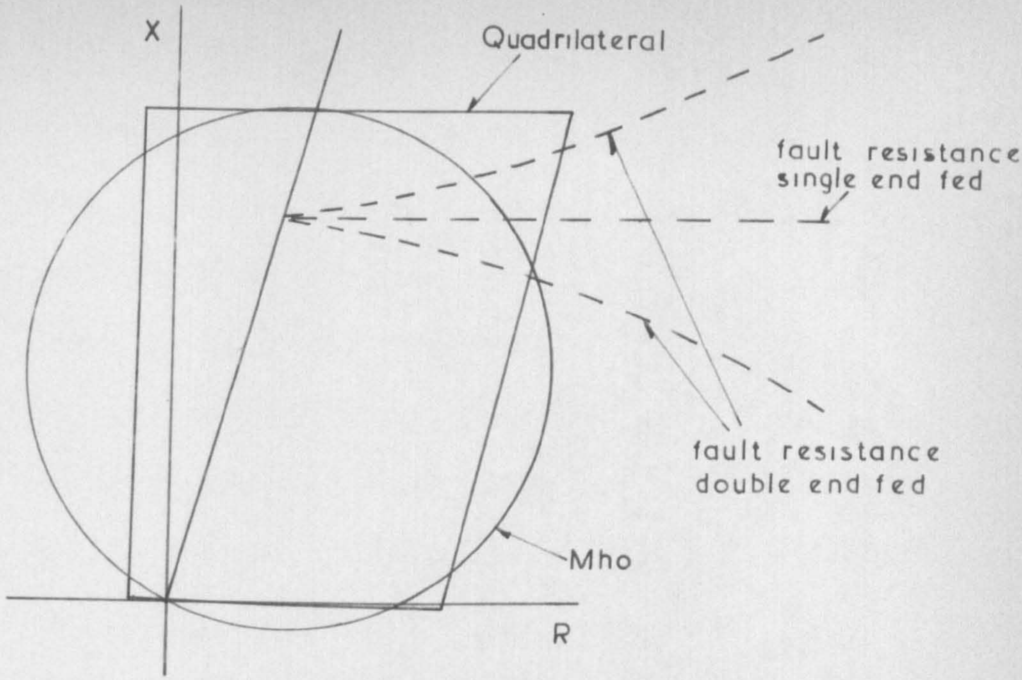


Fig.4.6 Loci of Fault Impedance with Resistive Faults Showing Coverage of Mho & Quadrilateral Relay

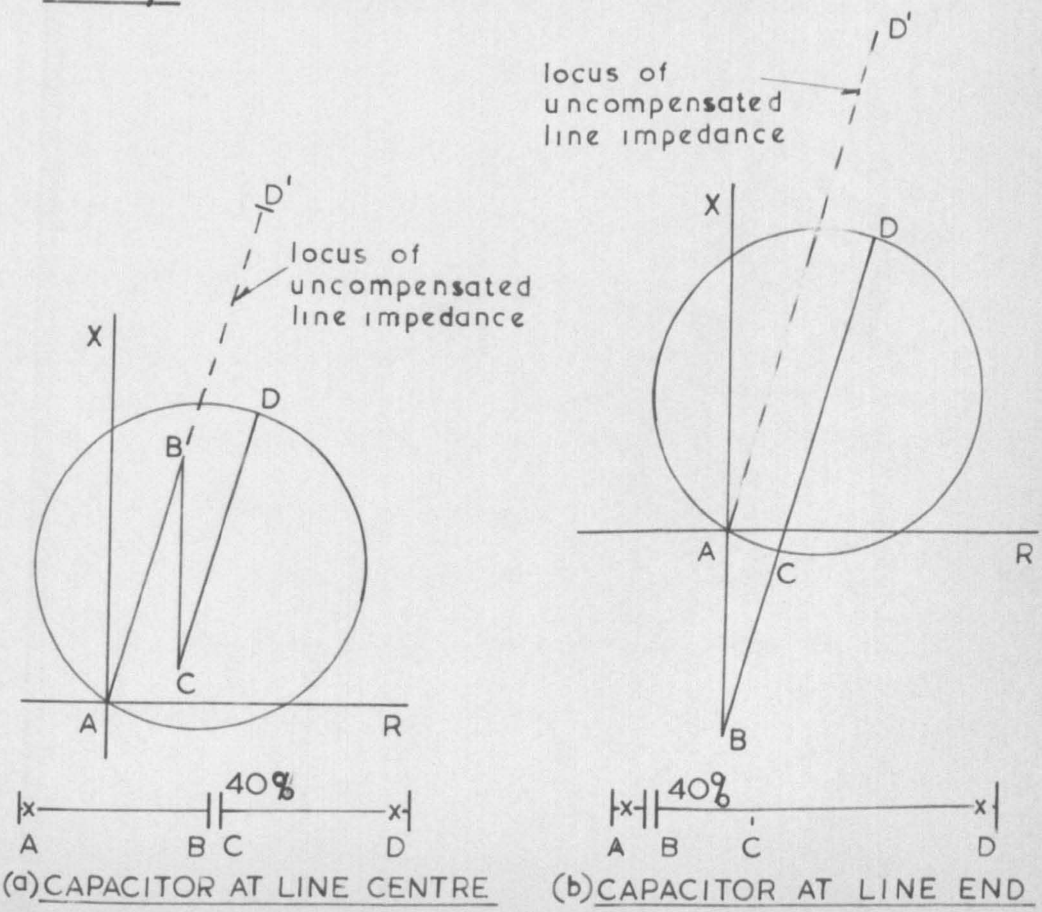


Fig.4.7 Impedance Loci of Series-compensated Lines

beyond a series capacitor. The effect of this discontinuity on the line impedance locus is shown in Fig. 4.7 for the two cases, capacitor at line centre and capacitor at relaying end of line. In the first case a mho characteristic can be obtained to cover the length of line required with certain allowance available for fault resistance and other inaccuracies. In the second case not only would the relay be blind to certain faults e.g. those in positions ABC but there is a danger that the relay arranged to protect the adjoining line might see these faults and cause erroneous circuit breaker tripping. However, it should be noted that these faults, although outside the particular characteristic shown, are within the line ABCD. The direction of power flow is, therefore, into the line and, in theory at least, a power flow direction relay should be able to assist in identifying the blind area. If in fact the relaying is by mho relays with sound phase polarising, then the adjacent line relay will only see forward faults and will not operate for faults along ABC providing that the particular fault is unbalanced.

The next complication is caused by the series capacitor's protective spark gap; depending upon system conditions and fault position the spark gap may or may not short circuit the capacitor. With the capacitor short circuited and with the settings chosen as in Fig. 4.7, only 60% of the line impedance AD is covered by the relay characteristic. It may be assumed that, for faults in the line section CD, the capacitor is in circuit prior to fault and if the capacitor is to be subsequently short circuited by the spark gap then this will occur, for most system conditions, in the first 5-20 ms. This is the very time when the distance relay is designed to make measurements and the short circuiting of a capacitor can introduce considerable transients into the relaying current and voltage. When the spark gap is of the extinguishing type, the capacitor can be reinserted into the transmission line very rapidly and, if the fault persists, will be rapidly removed again. It has been shown in section 3.3.3 and reference 34 that this process can result in a sustained offset in the relaying current. Up to the present time relay designs have relied upon impedance measurement under these conditions and it has been assumed that the particular value of impedance measured will lie part way between that expected with the capacitor in circuit and that expected with the capacitor out of circuit. The situation is far from ideal, but there are many relaying schemes in operation throughout the world attached to series compensated lines; all schemes are based upon impedance measurement and they do seem to give satisfactory service. Although it should be remembered that data regarding protective system failures are very slow to accumulate and have not yet been reported in the technical journals.

Some of the schemes in operation for the protection of these lines are described in the references.[6,7,10,23,41] With the exception of the early 220 kV Swedish lines where series capacitor installations were deliberately sited at line centres and the degrees of compensation were kept below 50%, all installations use some form of carrier or other communication channel to assist in identifying faults. In one particular installation both carrier and microwave links are used. The practice in America is to use either phase comparison protection using the communication channel(s) to carry information regarding current phase or else to use conventional distance relays with settings corresponding to the uncompensated line. This relies upon the spark gap operating and thus removing series capacitor(s) soon after fault occurrence, thus permitting correct relay operation. In some installations the spark-over voltage of the spark gaps have actually been reduced from the usual 2.5 - 3.5 p.u. range of settings in order to ensure capacitor short circuiting for all pertinent faults. The air blast type of extinguishing spark gap action is more suited to the distance protection than the magnetic type of spark gap because the air blast action takes longer to extinguish the arc initially and therefore allows more time forrelay operation. In Sweden where some of the 400 kV lines are over 300 miles in length, plain distance relays are used more frequently and use is made of carrier for blocking and tripping functions.

Nowhere have plain distance relays been used where there is some doubt as to whether a fault loop will be inductive or capacitive.

4.1.3 Distance Protection Errors in Series Compensated Lines

The basic error of all distance protection based upon impedance measurement is that the ratio V/I, varies between 0 and $\pm\infty$ during an a.c. cycle. Only when RMS values of V and I are taken, is the ratio equal to impedance. Since RMS value is only clearly defined for steady-state wave-forms, impedance is a rather nebulous term under transient conditions. Now, the operating time of modern distance relays is usually between one half and two cycles i.e. 10 ms. to 40 ms. In this period of time, not only are transient components of relaying wave-forms often significant, but it is also very difficult, if not impossible, to isolate the transient components. The waveforms of relaying quantities for series compensated lines have been produced using digital computer techniques, as described in chapter 3, and it has been shown that the transient component of series compensated line fault current is very different from uncompensated line fault current. Assuming no spark gap operation, the former is composed of a steady-state alternating part and a decaying transient alternating part. The latter is composed of a steady-state alternating part and a decaying unidirectional part. The relaying transient in uncompensated lines tends to produce relay over-reach. This is not the case for the compensated line. This is explained for the plain impedance relay as follows :

For operation $|Z| < |Z_N|$ where Z is measured impedance and Z_N is relay setting.

since $Z = V_{RMS}/I_{RMS}$, the operating condition can be written:

$$I_{RMS} > V_{RMS}/|Z_N| \quad .. \quad .. \quad .. \qquad 4.1$$

If V_{RMS} is constant before and during fault i.e. source impedance is small, then operation depends upon I_{RMS} only. The two cases for consideration are :-

a) An uncompensated line.

A direct component can only increase the RMS value of a current, since

$$I_{RMS} = \sqrt{I^2_{dc} + I^2_{ac}}$$

where I_{dc} is the average value of the direct component of I taken over, say, 10 ms. and I_{ac} is the RMS value of the alternating component of I. The relay, therefore, tends to overeach because of equation 4.1.

b) A series compensated line:

If it is assumed that load current is less than fault current, no direct component exists and the transient alternating component adds to the steady-state alternating component in such a way that the resultant wave-form of current builds up with time from the pre-fault load level to the steady-state fault level value. The value of I_{RMS}, taken over the first few successive intervals of say 10 ms. is, therefore, smaller than the RMS value of the steady-state fault current. The high speed impedance measuring distance relay, therefore, tends to underreach.

Another source of error in distance relaying is the effect of currents flowing in adjacent phases and circuits. The use of sound phase compensation is aimed at allowing for mutual coupling between phases of the same circuit but has been shown to be in error for long transmission lines. Bruce[34] showed that these errors can be made larger or smaller by the presence of a series capacitor.

In view of all these factors a different distance protection principle more suitable for series compensated lines should be sought. Some possible new methods are subsequently considered.

4.2 Choosing a new distance protection principle

Using the distance protection principle which will subsequently be chosen, a relay should be able to determine whether or not a fault is present within a given section of line from a consideration of the relaying voltages and currents. These quantities, obtained from the instrument transformers at one end of a line cannot be used to determine accurately the position of a fault because they are affected by the behaviour of the load and generation at the other line end and also by the condition of adjacent circuits. The objective of the distance relay is therefore to determine, within a certain maximum error, the position of a fault. The search for a new principle was aimed initially at the problem of protecting a single phase transmission line. Firstly the travelling wave phenomena was examined for possible application in a distance relay and secondly, the possibility of using the 'lumped parameter' differential equation of a fault loop was considered. The latter was thought to have possibilities and a particular method was chosen for further investigation.

4.2.1 Use of Travelling Wave-fronts for Measuring Distance to Fault

The use of travelling wave-fronts as a basis of a protective scheme is probably impracticable. Nevertheless their use should be considered.

Consider a wave-front which starts from the sending end and travels along a line. If it is confronted by an open or short circuit or another type of discontinuity, then all or part of the wave-front will be reflected back towards the sending end. Thus the sending end current and voltage may be modified after a minimum of twice the time taken for a wave-front to travel to the discontinuity. Since wave-fronts are propagated along aerial lines at a velocity approaching the speed of light in air, it has been suggested that the time delay between modifications of the sending end current and voltage could be used as a measure of fault position since :-

distance to fault = velocity of propagation x time delay x 0.5.

Before the principle can be applied to complex systems containing multiphase lines, many generators and loads etc., the problem of measuring fault position on a single phase, single generator fed series compensated line is considered. It is assumed that the line is initially unenergised, the source impedance is small and the line circuit breakers are closed with a fault located at a distance from the generator. The relaying wave-forms are shown in Fig. 4.8 and Fig. 4.9 for a fault located at a distance of 230 miles along such a line. The wave-forms in Fig. 4.8 correspond to the line circuit breakers closing at a peak of the generator voltage. The relaying voltage is almost a pure sine wave since source impedance is small. The relaying current, I_s on the other hand, is very discontinuous. Initially $I_s = E_s/Z_c$ and this wave-front travels along the line until it meets a discontinuity at the fault. The appearance of the wave-front at the capacitor after a time delay and then its appearance at the fault after a further delay are clearly shown in the figure. At the fault, the current must suddenly change, unless the fault impedance = Z_c, and this discontinuity appears in the relaying current after a further time delay. As can be seen in the wave-form of I_s, the current contains discontinuities, regularly spaced in time. When the capacitor spark gap operates, extra travelling waves are introduced and the overall picture becomes rather complicated. In the circuit for which Fig. 4.8 shows the wave-forms, only a small amount of lumped inductance is present and the travelling wave details are therefore not camouflaged to a great extent. As a result the discontinuities caused by the travelling waves reacting with the fault can be separated from each other. Inspection of pI_s, the derivative of I_s, shows this most clearly: the regular beat due to the fault is separable from the effect of

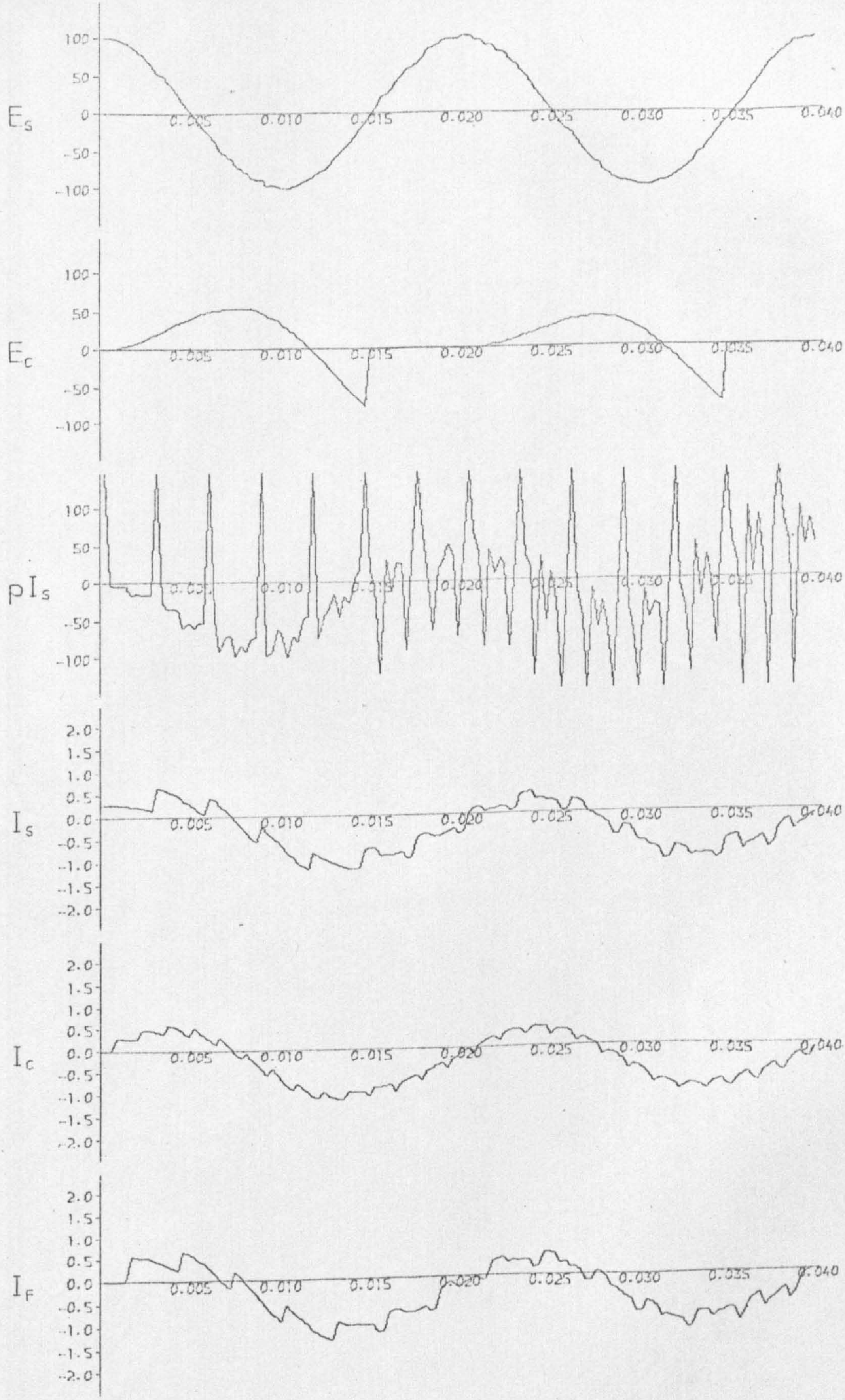


Fig.4.8 Wave-forms following energisation of line with fault present. Line switches closed at voltage maximum. See Fig. 3.15 for circuit constants

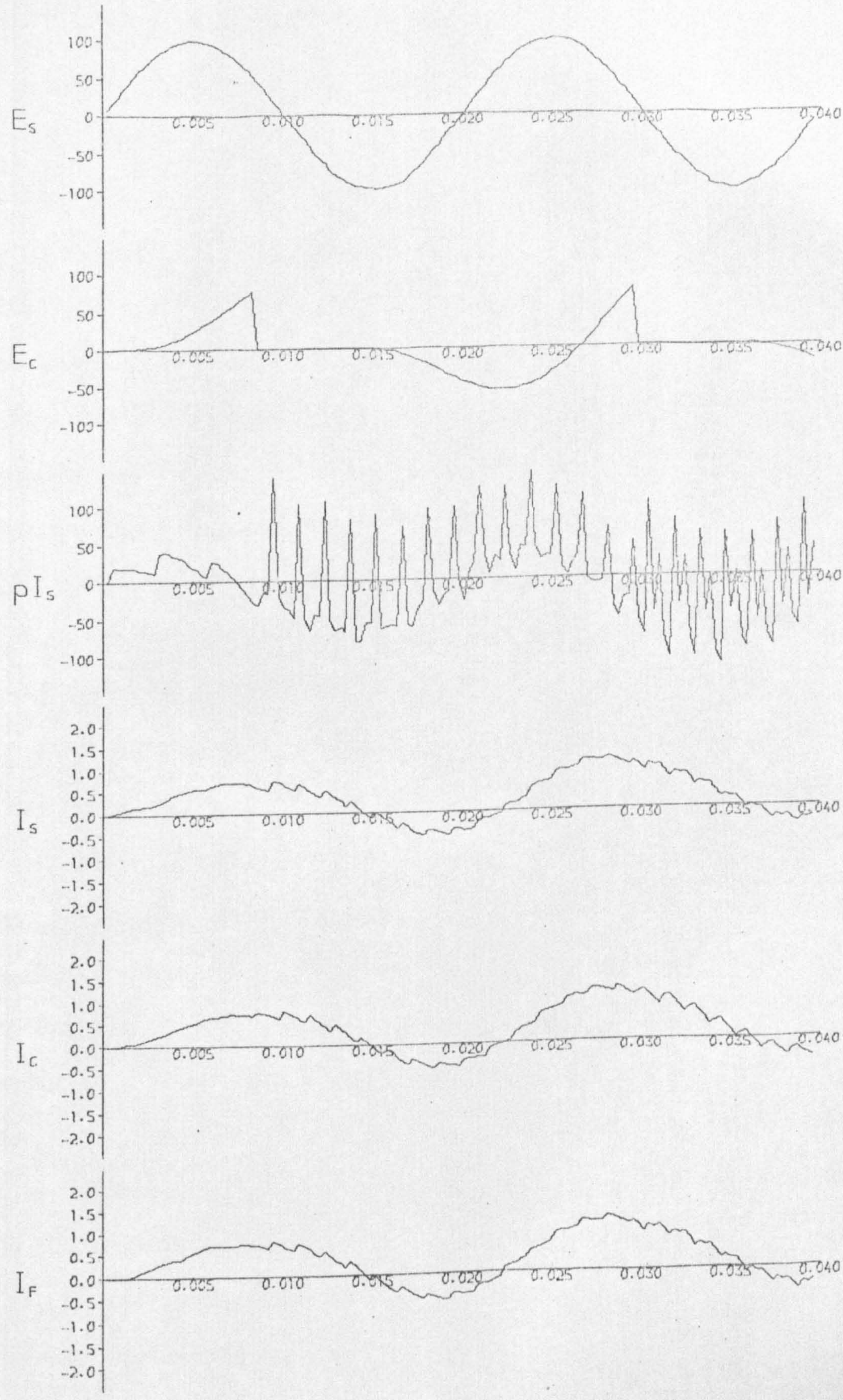


Fig. 4.9 Wave-forms following energisation of line with fault present. Line switches closed at voltage zero. See Fig. 3.15 for circuit constants.

the spark gap operation. At this stage it can be said that if the relaying quantities are always of the form shown in Fig. 4.8 then a practical distance measuring scheme could possibly be developed. Unfortunately faults can and do occur under different conditions. Wave-forms are shown in Fig. 4.9 for an identical circuit to that considered above, but the line circuit breakers are closed at a voltage zero instead of at a voltage maximum. The current is not now discontinuous, at least not until the spark gap operates, although the first derivative of current is discontinuous. Any equipment which relies on the second derivative would have great difficulty in discerning between current jumps due to travelling waves and current jumps due to spark gap operation and if the relaying current in Fig. 4.8 is differentiated twice then small discontinuties in the first derivative caused by the build up of voltage on the capacitors would be amplified also. To further complicate the situation an actual series compensated transmission line would be at least 100 miles in length and would almost certainly travel over very different terrain. This would cause further distortion of the sending current due to a variation of line impedance with line length and this would be most noticeable in the derivatives of current. With the added effects of mutual coupling between phases and between circuits, the situation becomes too complicated and is not considered further.

The possibility of injecting a coded high frequency signal into the line and thereby being able to discriminate between reflections of the injected signal and other travelling waves should not be dismissed so readily. There are two serious problems however. Firstly, the problem of mutual coupling between phases can mean that high frequency signals injected along phase A can appear at the other end of the line on phase B. Secondly, the effect of high impedance faults equal in value to the characteristic impedance of the line (about 330 Ω) would cause no reflections and the fault would not be recognised. These high fault impedances are possible, the two most common being associated with broken or sagged line faults in dry regions and the so-called vegetation faults which are particularly associated with tropical regions.

4.2.2 Choosing a protective scheme based upon lumped parameter representation of fault loop.

If the line susceptance of a series compensated transmission line is neglected, then the fault loop is a simple R, L, C series circuit and the differential equation is:

$$R.i + L\frac{di}{dt} + \frac{1}{C} . \int_{to}^{t} idt + V_c(t_o) = v \quad .. \; .. \quad 4.2$$

i and v are the relaying current and voltage R, L and C are unknown, V_c is the capacitor voltage. The value of L is the line inductance between the relay point and the fault and therefore represents the distance of the fault from the relay. A number of methods are available for measuring L. Each scheme is considered in turn:

Scheme 1 Differentiate equation 4.2 three times and substitute $p = \frac{d}{dt}$ etc. thus :

$$\begin{bmatrix} pi & p^2i & i \\ p^2i & p^3i & pi \\ p^3i & p^4i & p^2i \end{bmatrix} \times \begin{bmatrix} R \\ L \\ \frac{1}{C} \end{bmatrix} = \begin{bmatrix} pv \\ p^2v \\ p^3v \end{bmatrix}$$

and hence solve for L. This value of L can be determined continuously i.e. as a function of time and since fault position is proportional to L, a distance relay can be properly based. The disadvantage of the scheme is that the derivatives, especially the higher order derivatives, can introduce considerable errors due to noise and distortion in the i and v signals and this scheme is therefore not practicable.

Scheme 2

Differentiate equation 4.2 twice and assume that inductance and resistance up to fault are proportional i.e. $R = kL$. This assumption is reasonable for solid faults since both R and L are functions mainly of the transmission line geometry, with some dependence upon frequency because of skin effect and earth currents and a small dependence upon voltage because of corona effect. The penalty of introducing the assumption is that error is introduced for resistance faults but all distance schemes can suffer in this way. The extra error is counter-balanced by some simplification in the mathematical base with consequent potential saving in hardware in a practical relay. The equation is:

$$\begin{bmatrix} k.pi + p^2 i & i \\ k.p^2 i + p^3 i & pi \end{bmatrix} \times \begin{bmatrix} L \\ \frac{1}{C} \end{bmatrix} = \begin{bmatrix} pv \\ p^2 v \end{bmatrix}$$

hence can solve for L. The great disadvantage is that the higher order derivatives introduce considerable problems with noise and distortion. This scheme has been investigated by Chidolue.[42]

Scheme 3

The value of the capacitor voltage V_c can be determined at any instant if the fault loop parameters are known together with the relaying voltage and current hence:

$$V_c(t_0) = v(t_0) - R.i(t_0) - L.pi(t_0)$$

also

$$V_c(t_1) = v(t_1) - R.i(t_1) - L.pi(t_1)$$

$$V_c(t_2) = v(t_2) - R.i(t_2) - L.pi(t_2)$$

These values for V_c can be substituted into equation 4.2 in order to obtain:

$$\begin{bmatrix} i(t_1) - i(t_0) & pi(t_1) - pi(t_0) & \int_{t_0}^{t_1} i\,dt \\ i(t_2) - i(t_1) & pi(t_2) - pi(t_1) & \int_{t_1}^{t_2} i\,dt \\ i(t_3) - i(t_2) & pi(t_3) - pi(t_2) & \int_{t_2}^{t_3} i\,dt \end{bmatrix} \times \begin{bmatrix} R \\ L \\ \frac{1}{C} \end{bmatrix} = \begin{bmatrix} v(t_1)-v(t_0) \\ v(t_2)-v(t_1) \\ v(t_3)-v(t_2) \end{bmatrix}$$

As before this equation can be solved for L.

The advantage of using this method is that data is used from several instants of the relaying wave-forms and the value of the definite integrals required are relatively free from noise and distortion in the relaying wave-forms.

There are two major disadvantages. Firstly, if any of the fault loop parameters change during the period t_0 to t_3 then the resulting value of L is in error. Such a change of parameters may be caused by a shorting of the capacitor by its own protective spark gap or the occurrence of a second fault on the system. Secondly, the need for the difference between two first derivatives introduces similar problems to the schemes where a second derivative is required.

Scheme 4

This scheme is identical to Scheme 3 except that the simplifying assumption of R = kL is made, thus enabling a much simpler equation to be derived:-

$$\begin{bmatrix} R(i(t_1)-i(t_0))+pi(t_1)-pi(t_0) & \int_{t_0}^{t_1} i\,dt \\ R(i(t_2)-i(t_1))+pi(t_2)-pi(t_1) & \int_{t_1}^{t_2} i\,dt \end{bmatrix} \times \begin{bmatrix} L \\ \frac{1}{C} \end{bmatrix} = \begin{bmatrix} v(t_1)-v(t_0) \\ v(t_2)-v(t_1) \end{bmatrix}$$

which can be solved for L.

In comparison with Scheme 3 the equation is simpler and the time period during which a circuit change can cause error is shorter i.e. $t_0 - t_2$ as against $t_0 - t_3$. However, the difference of two derivatives is required and the implications of assuming R and L proportionality which are discussed under Scheme 2, also apply.

Scheme 5

As in Scheme 4 expressions for V_c and R are substituted into equation 4.2 hence :-

$$(k(i(t)-i(t_0))+pi(t_1)-pi(t_0)).L+\int_{t_0}^{t} idt.\frac{1}{C} = v(t)-v(t_0)$$

and $$(k(i(t)-i(t_1))+pi(t)-pi(t_1)).L+\int_{t_1}^{t} idt.\frac{1}{C} = v(t)-v(t_1)$$

integrating the first equation over the period t_0 - t_1 and the second equation over t_1 - t_2 another equation set is obtained :-

$$\begin{bmatrix} k\int_{t_0}^{t_1}(i(t)-i(t_0))dt + i(t_1)-i(t_0) - \Delta t.pi(t_0), & \int_{t_0}^{t_1}\int_{t_0}^{t} idtdt \\ k\int_{t_1}^{t_2}(i(t)-i(t_1))dt + i(t_2)-i(t_1) - \Delta t.pi(t_1), & \int_{t_1}^{t_2}\int_{t_1}^{t} idtdt \end{bmatrix} \times \begin{bmatrix} L \\ \frac{1}{C} \end{bmatrix}$$

$$= \begin{bmatrix} \int_{t_0}^{t_1} vdt - \Delta tv(t_0) \\ \int_{t_1}^{t_2} vdt - \Delta tv(t_1) \end{bmatrix}$$

where $\Delta t = t_2 - t_1 = t_1 - t_0$.

The advantage of this scheme over scheme 4 is that the equation contains more definite integrals and less current derivatives. However, the expressions: $i(t_2)-i(t_1)- \Delta t.pi(t_1), \int_{t_1}^{t_2} vdt - \Delta tv(t_1)$ etc., are frequently very small unless Δt is made fairly large i.e. greater than about one quarter of the period of the fundamental cycle.

Scheme 6

If time, t_o , is an instant in the steady state when the sinusoidal current is a maximum then $V_c(t_0) = 0$ and equation 4.2 becomes:-

$$R.i + L\frac{di}{dt} + \frac{1}{C}.\int_{t_o}^{t} idt = v$$

with the assumption R = k.L, as in Scheme 2, then

$$(k.i + \frac{di}{dt})L + \frac{1}{C}.\int_{t_o}^{t} idt = v \quad .. \quad .. \quad .. \quad .. \quad 4.3$$

This equation can be written down for two different values of t to form the required equation for the evaluation of L. Hence:-

$$\begin{bmatrix} k.i(t_1) + \frac{di}{dt}(t_1) & \int_{t_0}^{t_1} idt \\ k.i(t_2) + \frac{di}{dt}(t_2) & \int_{t_0}^{t_2} idt \end{bmatrix} \times \begin{bmatrix} L \\ \frac{1}{C} \end{bmatrix} = \begin{bmatrix} v(t_1) \\ v(t_2) \end{bmatrix}$$

The first derivative of current has thus been successfully isolated and the form of the equation for L is not so complicated as with other schemes. A system of fixing t_0 fairly frequently is required such that drift in the evaluation of $\int_{t_0}^{t} idt$ is prevented from introducing error.

Scheme 7

Since it is desirable to remove the dependence of a distance relay on the derivatives of current or any other quantity the equation 4.3 can be integrated over the periods $t_1 - t_2$ and $t_2 - t_3$ thus obtaining:

$$\begin{bmatrix} k\int_{t_1}^{t_2} idt+i(t_2)-i(t_1) & \int_{t_1}^{t_2}\int_{t_0}^{t} idtdt \\ k\int_{t_2}^{t_3} idt+i(t_3)-i(t_2) & \int_{t_2}^{t_3}\int_{t_0}^{t} idtdt \end{bmatrix} \times \begin{bmatrix} L \\ \\ \frac{1}{C} \end{bmatrix} = \begin{bmatrix} \int_{t_1}^{t_2} vdt \\ \int_{t_2}^{t_3} vdt \end{bmatrix}$$

the equation for L is then:- $L =$

$$\frac{\int_{t_1}^{t_2} vdt.\ \int_{t_2}^{t_3}\int_{t_0}^{t} idtdt \ - \ \int_{t_2}^{t_3} vdt.\ \int_{t_1}^{t_2}\int_{t_0}^{t} idtdt}{(k\int_{t_1}^{t_2} idt+i(t_2)-i(t_1)).\int_{t_2}^{t_3}\int_{t_0}^{t} idtdt-(k\int_{t_2}^{t_3} idt+i(t_3)-i(t_2)).\int_{t_1}^{t_2}\int_{t_0}^{t} idtdt} \qquad \text{.. .. 4.4}$$

if the assumption $R = kL$ had not been made then the expression for L would have been dependent upon a 3 x 3 matrix and its complexity would have been considerably increased as a result. The abundance of definite integrals in the equation for L makes for better immunity against noise and distortion in the relaying wave-forms. As with the other schemes which take data over a period of time, any change in the circuit condition during this time is liable to cause error in the evaluation of L.

The differences $i(t_2) - i(t_1)$ and $i(t_3) - i(t_2)$ are related to the derivative of current by the formula

$$i(t_2) - i(t_1) = (t_2 - t_1).\ \frac{d}{dt}i((t_1+t_2)/2)$$

$$(t_2 - t_1) \rightarrow 0$$

Providing that $t_2 - t_1$ is not small the disadvantages of using derivatives are not introduced into the scheme.

It was decided to investigate Scheme 7 with a view to eventually building a distance relay.

4.3. Considerations of the chosen scheme, a sampling relay

On the basis of the points made in section 4.2.2, it was decided to investigate further the possibility of constructing a distance relay based upon equation 4.4. Since values of current are required to be known at different instants of time, the relay can be called a Sampling Relay.

The first consideration is the possibility of there being no transient in the relaying waveforms. Clearly, if the line current and voltage are in the steady-state then it is not possible to determine the amount of inductive reactance in an R-L-C circuit because the observed phase angle between the two signals is determined by the ratio of net circuit reactance to circuit resistance. In fact, in the scheme being considered a proportional relationship has been assumed between R and L and, therefore, the inductance can be measured by determining the circuit resistance and multiplying this by the constant of proportionality. Unfortunately, this factor is somewhat variable with fault conditions as described in section 4.1 and resistance measurement cannot, therefore, be relied upon for accurate distance relay operation. The conditions for no transient are examined in section 4.3.1 and it is concluded that transient free wave-forms cannot be produced in an R-L-C circuit providing that prior to line energisation, the series capacitor is not charged to a high voltage.

The other concern is the response of the Sampling Relay when the relaying waveforms are distorted due to travelling waves and other effects. Considered first is the response of a relay designed to measure perfectly fault position in a simple R-L-C circuit. Secondly the response of a more practical Sampling Relay, designed with some inherent immunity against distorted waveforms, is given some attention.

4.3.1 Conditions for No Transient in R-L-C Circuit

The equation for the current in a simple series R-L-C circuit, energised by a voltage $e = E\cos(\omega t+\theta)$ has already been encountered in section 3.2.

The equation for current is:-

$$i = e^{-\alpha t}.(A\cos\beta t + B\sin\beta t) + E\cos(\omega t + \theta - \varphi)/Z$$

α, β, φ and Z are functions of the circuit constants R, L and C.

A and B depend upon initial conditions and for no transient in i and hence in a relaying voltage, $A = B = 0$. The value of switching angle, θ and capacitor voltage, V_c can be determined for this condition with $t = 0$.

The pertinent equations are :-

$$i(0) = A + E\cos(\theta - \varphi)/Z \quad .. \quad .. \quad 4.5$$

$$pi(0) = -\alpha A + B\beta - E\sin(\theta - \varphi).\omega/Z \quad .. \quad 4.6$$

and also $pi(0) = (E\cos\theta - i(0).R - V_c(0))/L \quad .. \quad 4.7$

from equation 4.5 the condition for A = 0 can be obtained, providing that $i(0) \not> E/Z$. In general two values of θ, θ_1 and θ_2 satisfy the condition.

The equation for B, with A = 0, obtained by subtracting equation 4.7 from equation 4.6 yields the necessary condition for B = 0. It is :-

$$V_c(0) = E\cos\theta - i(0).R + \omega L.E\sin(\theta - \varphi)/Z$$

and this is the equation for capacitor voltage with the R-L-C circuit in question operating in the steady-state with $t = 2n\pi/\omega$. Thus there are two conditions which are necessary for there to be a transient free current. The first is that the fault occurs at one of the particular points-on-wave when the initial current flowing is equal to the steady-state current at that instant. The second is that the value of capacitor voltage at these instants corresponds to the steady-state current which would flow for the particular point-on-wave appertaining. These conditions can never be satisfied if the fault on a transmission line occurs whilst the system is loaded and if the line switches are closed on to a fault, it would be necessary for the capacitor voltage to equal the peak value associated with the anticipated steady-state fault current for there to be no transient. This latter condition is most unlikely to be satisfied since it is not normal practice to energise transmission lines which are charged to a high voltage without first reducing the amount of charge, the reason for this being that unwanted high amplitude switching surges are dependent upon line-side voltage at the instant of line energisation.

4.3.2 Performance of ideal sampling relay with practical wave-forms

Having chosen to investigate the performance of a Sampling Relay it is necessary to consider the effectiveness of the equipment when required to deal with practical wave-forms.

The relaying current can be represented by a Fourier series :-

$$i(t) = \sum_{n=1}^{\infty} A_n . \sin(n\omega t + \alpha_n)$$

the higher values of n are mainly associated with the effect of travelling waves. The definite integrals of this signal which are required in the equation 4.4, for measurement of fault position are then of the form:-

$$\int_{t_1}^{t_2} i(t)dt = \sum_{n=1}^{\infty} \frac{A_n}{n\omega}[\cos(n\omega t_1 + \alpha_n) - \cos(n\omega t_2 + \alpha_n)]$$

Clearly the value of this integral which involves the reciprocal of n is weighted in favour of the low frequencies i.e. the low values of n. This means that the definite integral is relatively free from errors caused by travelling waves.

The differences in instantaneous values of the current which occur in equation 4.4 can be related to the derivative of the current by an approximate formula :-

$$i(t_2) - i(t_1) \doteqdot (t_2 - t_1).\frac{di(t')}{dt}; \; t' = (t_2 + t_1)/2.$$

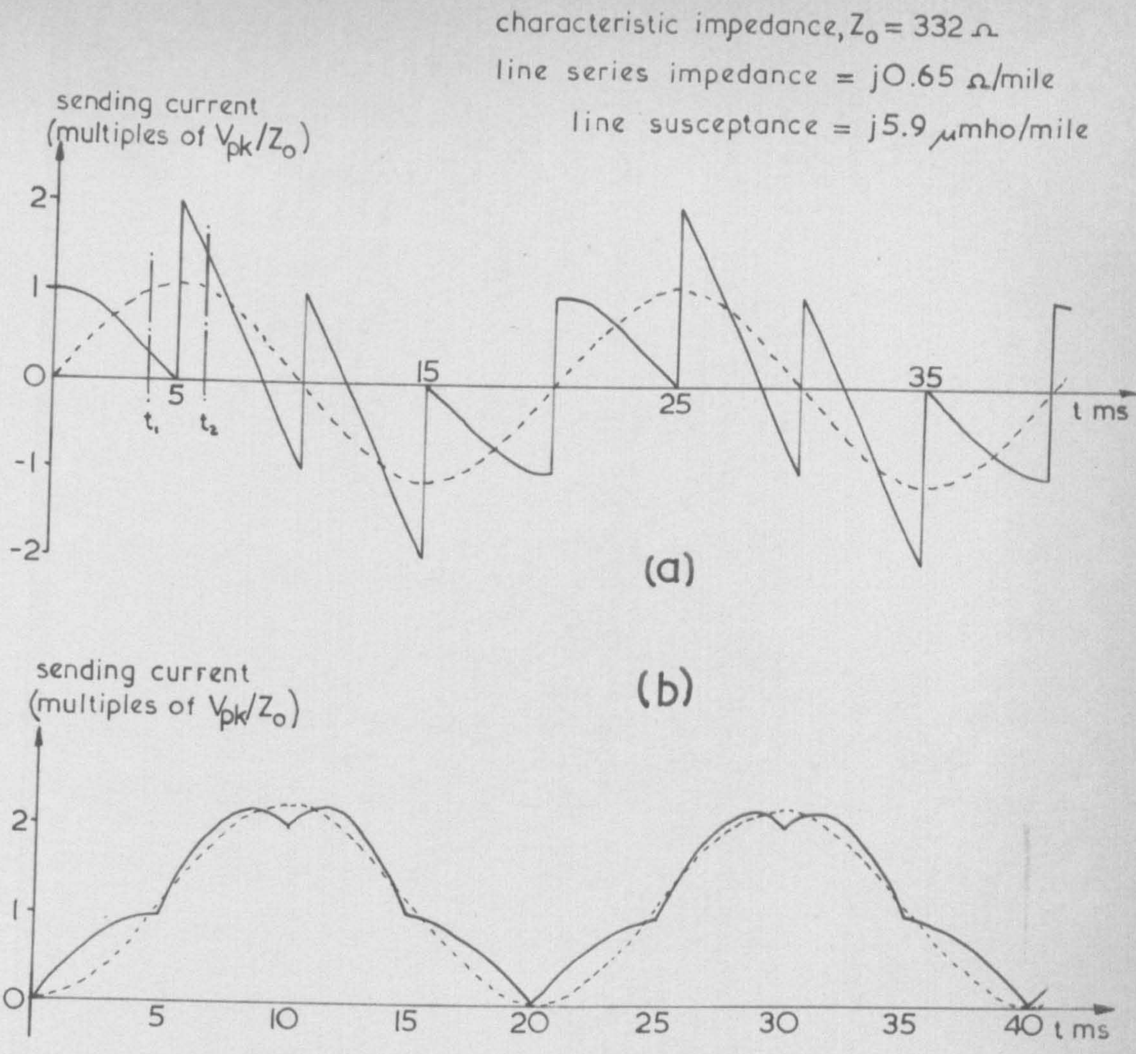
By considering the trigonometric series for the derivative it is apparent that the difference of currents is very dependent upon the high frequencies. This important point is perhaps better appreciated by considering the practical wave-form of Fig. 4.10a. If instantaneous values of current are taken at t_1 and t_2 then the difference $i(t_2) - i(t_1) = 5$ units, whereas the true difference which is required for the measuring scheme to be effective is 0 units. These very large errors in current differences are the cause of such large errors in the measurement of fault position that it must be concluded that 'perfect' equipment is too susceptible to irregularities in the wave-forms to be practicable.

4.3.3 A practical sampling relay - design considerations

There are four basic requirements of the sampling relay based upon equation 4.4:

1. With a fault at the setting of the relay, the measurement error due to inherent weaknesses in the measuring techniques should not exceed 10 per cent of the setting.

2. The time taken to complete measurement should be less than 30 ms.



KEY

Waveform when line susceptance is taken into account; surges traverse the line

Waveform when line susceptance is neglected; no travelling waves

Fig. 4.10 Current in a Loss-less Transmission Line Fed by an Infinite Source. Line Energised when:-

(a) Source Voltage is at a Peak, V_{pk}

(b) Source Voltage is Zero, Going Positive

A Solid Fault is Present 465mls from Source

3. The equipment must be able to manage the range of signal amplitudes which occur in the system.

4. The equipment reliability must be acceptable.

The first two of these requirements are given some attention in this chapter. The third point is dealt with in chapter 5.1 and the fourth point is mentioned in chapter 6.

4.3.3.1 Modifications to Ideal Sampling Relay

It has been demonstrated in section 4.3.2 that the measuring technique can be hopelessly in error if the value of the relaying current $i(t_1)$ in the equations is taken as being the value of the relaying current at the instant, t_1. This is a major weakness in the technique, and a method of eliminating or at least of reducing the dependence of $i(t)$ on the travelling waves was therefore sought. Analogue and digital filters are effective in removing distortion but phase shift and time delay associated with the filters have to be carefully considered. Digital filters, in particular, may take a considerable time to produce a result and this has to be taken into account when designing an overall protective scheme which is to operate in a short time. It was therefore decided to investigate the use of a very simple but fast technique of reducing the effect of travelling waves. The method may be realised by analogue circuitry or else by digital computer program. The simple analogue circuitry required is shown in Fig 4.11 and its operation is as follows :

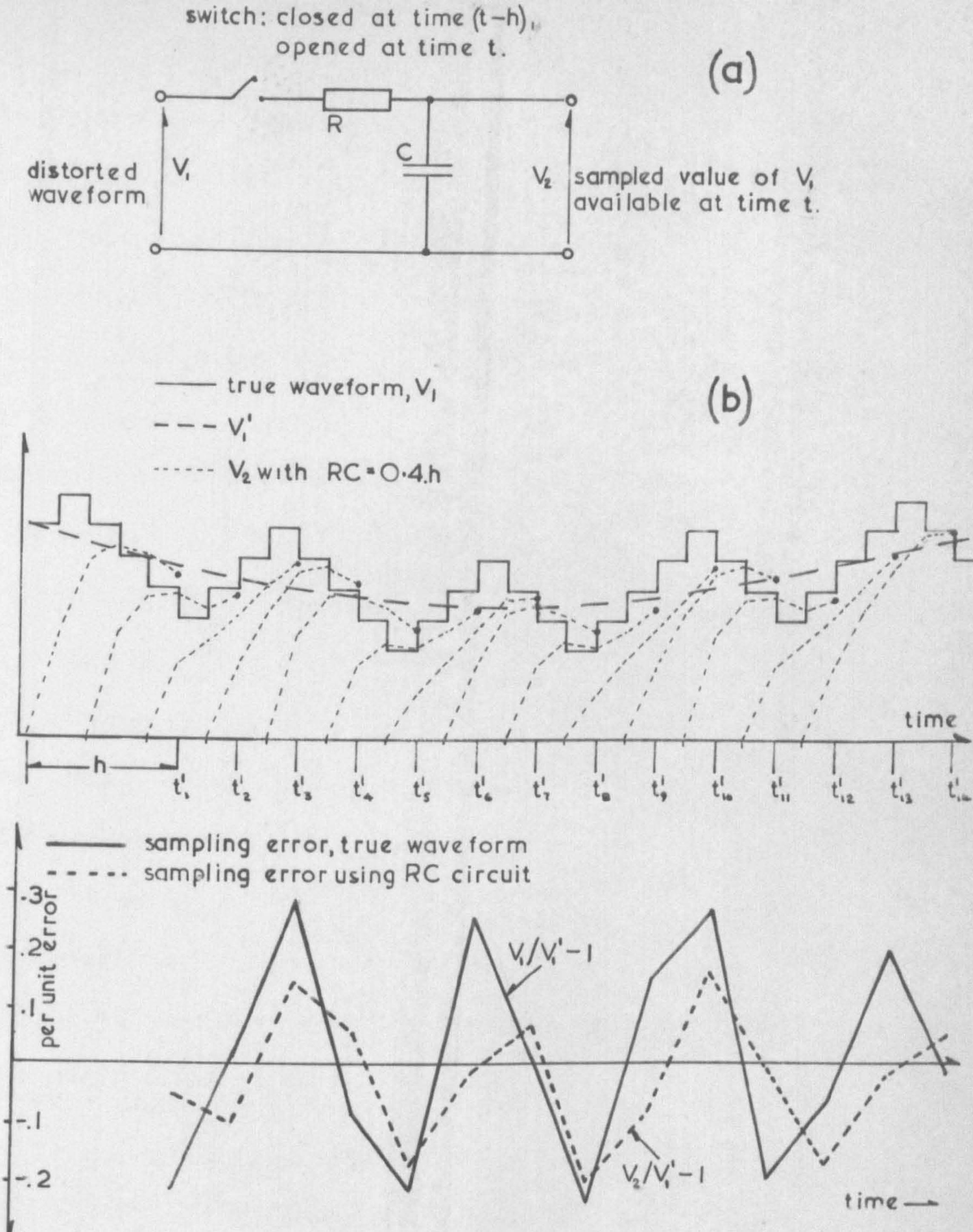


Fig.4.11 Effect of Switched RC Circuit in Sampling Distorted Waveform
(a) The Circuit
(b) Circuit Voltages V_1 & V_2 against Time also Sampling Error.

Suppose the distorted wave-form of Fig. 4.12 is available and samples of the underlying trend in the wave-form are required at instants t'_1, t'_2, t'_3 .... The circuit of Fig. 4.11 is fairly effective in detecting these samples. Its performance depends upon the time constant, RC, of the circuit and the instant of closing the switch. If the switch is closed at time t - h, the sampled value is available at time t, i.e. there is no time delay. An arbitrary time constant and an arbitrary value of h has been chosen to illustrate the performance of the circuit. The error associated with the sampling is shown in Fig. 4.13 alongside the error corresponding to a perfect sampling of the actual wave-form. The voltage on the capacitor at times greater than t represents the sampled value of the wave-form at time t. If the circuit which subsequently uses this voltage has a high input impedance then the capacitor must hold its charge for a long period. The configuration is a simple form of 'Sample and Hold' circuit and is used in the construction of the prototype sampling relay, described in chapter 5.

The digital computer program corresponding to the circuitry described is equally simple and almost as fast: if a distorted waveform, A (t), is to be sampled at time t and if the computer is fed with data regarding the value of A at intervals h/p i.e. fed with the values $A\ (t + \frac{nh}{p})$, n = ... -3, -2, -1, 0, 1, 2 ..., then the approximate sampled value of the undistorted wave-form can be calculated from the expression

$$\begin{array}{l}\text{sampled value of } A(t)\\ \text{at time } t\end{array} = \sum_{j=0}^{j=p} A\left(t - \frac{j.h}{p}\right).\left(1 - \frac{1}{m}\right)^{j}/m$$

m, j and p are integers and $\frac{1}{m}$ is proportional to the time constant, RC, of the corresponding analogue circuit. This algorithm which is derived in Appendix 4.1, accurately represents the analogue circuit for $\frac{h}{p} \ll RC$. It has been used extensively in the digital modelling of analogue circuitry in this thesis and it would be useful in a fully digitised protection scheme where fast digital filtering would be required.

4.3.3.2 Relay Operating Time Considerations

In choosing an operating time which is suitable for practical equipment, a number of factors should be taken into account. Since samples of current are required at instants in time t_1, t_2 and t_3 in equation 4.4 and integrals of current and voltage are required between these times, it is clearly impossible to design equipment which operates faster than $(t_3 - t_1)$ seconds. In fact, because of the errors described above it would be unwise to rely on a single calculation for the period $t_1 \rightarrow t_3$; many calculations should be performed i.e. for periods $t'_1 \rightarrow t'_3$, $t'_2 \rightarrow t'_4$, $t'_3 \rightarrow t'_5$, ... and relay operation would then be more reliably based. A minimum of five such periods would seem to be reasonable. The actual periods $t_2 - t_1$, $t_3 - t_2$ should not be too small because in the first place, the current differences $(i(t_2) - i(t_1))$ and $(i(t_3) - i(t_2))$ would be small and much in error for certain parts of the current wave-form and in the second place, the definite integrals of current and voltage would be substantially in error due to travelling wave distortion if the integration period does not encompass at least one discontinuity in the wave-forms. From a consideration of all these factors a step length of between 2.5 and 3 ms. would seem to be a good compromise and this would permit at least 5 calculations within 20 ms. Although for lines in excess of about 300 miles in length, wavefronts may take more than 1.5 ms. to travel from the fault to the relay and even a step length of 3 ms. could cause unacceptable errors. In such cases, a longer operating time may need to be tolerated.

4.3.3.3 Performance of Practical Relay Subjected to Practical Wave-Forms

A digital computer was used to check that satisfactory relay operation results when equation 4.4 is applied to the wave-forms produced in series compensated lines. From equation 4.4, restrain signal S_1 and operate signal, S_2 are derived, thus:

$$S_1 = \int_{t_1}^{t_2} v dt \quad x \int_{t_2}^{t_3} \int_0^t idtdt - \int_{t_2}^{t_3} vdt \quad x \int_{t_1}^{t_2} \int_0^t idtdt \qquad \ldots 4.8(a)$$

$$S_2 = L_N \times \left[\left(k \int_{t_1}^{t_2} idt + i(t_2) - i(t_1)\right) \times \int_{t_2}^{t_3} \int_0^t idtdt - \right. \qquad \ldots 4.8(b)$$

$$\left. \left(k \int_{t_2}^{t_3} idt + i(t_3) - i(t_2)\right) \times \int_{t_1}^{t_2} \int_0^t idtdt \right)\right]$$

where L_N is the relay setting.

In calculating the values of S_1 and S_2 from a given set of i and v wave-forms, it is assumed that the sampled values of the current are obtained by the method described in section 4.3.3.1 since the effect of travelling waves is then somewhat reduced. It is inevitable that this sampling process introduces errors into the relay when the wave-forms are free from travelling wave effects i.e. a fault loop of lumped parameters, and so the time constant of the sampling process must be a compromise chosen to reduce the errors due to travelling waves without introducing significant errors into undistorted wave-forms. After some preliminary analysis it was decided that a total sampling time of 5 ms. with a time constant of 2 ms. was a good compromise. Suitable values of p, m and h for use in the digital computer program are then 20, 8 and 0.25 ms. respectively.

In the calculations it is also assumed that the sampled values of current degrade by 3 per cent during the 'hold' time associated with the 'Sample and Hold' process. This would not occur if the sampled values were stored digitally but with analogue equipment some degradation is normally expected. The wave-forms for i and v which were used in evaluating the relay performance were obtained using the techniques described in section 3.3.

The two major factors which affect the performance of the sampling relay are travelling waves and protective spark gap operation. The first factor introduces errors mainly into the sampling of the current and the second factor results in erroneous measurement of line inductance when this is dependent upon data acquired during a period when the spark gap starts or ceases to operate. The method of overcoming these effects depends upon the actual form of the sampling relay equipment. Two equipment types are considered:-

a) digital equipment - binary numbers representing the values of S_1 and S_2 at a particular instant can be divided and the quotient, S_1/S_2, obtained. It is then a relatively simple matter to identify spurious results caused by spark gap operation.

b) analogue equipment - division of signals for S_1 and S_2 is not easily achieved. However, it is only necessary to decide whether or not the relay should operate. This can be determined by applying the comparisons,

$$|S_1| < |S_2| \quad ? \qquad \ldots \quad \ldots \quad \ldots \quad 4.9(a)$$

$$|S_1 + S_2| > |S_1 - S_2| \quad ? \qquad \ldots \quad \ldots \quad \ldots \quad 4.9(b)$$

For correct relay operation both results must be affirmative since the first comparison relates to the distance of the fault from the relaying point and the second to whether S_1 and S_2 are of the same algebraic sign. Spurious data should be removed, as far as possible, from the S_1 and S_2 signals before the comparisons are applied.

The figures 4.12 and 4.13 show the way in which a relay incorporating simple filtering features is able to deal with these basic errors. Fig. 4.12 corresponds to the sampling relay subjected to wave-forms obtained from a lumped parameter fault loop and Fig. 4.13 to the same equipment except that the relaying wave-forms were derived from a distributed parameter fault loop. The fault loop parameters are listed in Table 4.1.

In each of the figures the wave-forms of S_1 and S_2, calculated by the digital computer, are shown constant for a period of 3 ms. This is the basic time step chosen for the relay and is computed from data obtained during the preceding two time steps i.e. the preceding 6 ms. The quotient S_1/S_2 which should represent fault position in proportion to relay setting is shown to be a constant function with some random error except for the three values between 18 ms. and 27 ms. where the function is in greater error due to spark gap operation. In a numerical treatment of the problem it would clearly be possible to identify these spurious values and a reasonably accurate value of fault loop inductance obtained, whether or not travelling waves are present. In an analogue realisation of the sampling relay, the practical difficulties demand a different approach as explained above. The wave-forms for S_1 and S_2 are filtered using a simple R-C circuit and the resultant smoothed signals shown, are applied to a comparator yielding a logical result b_1 from equation 4.9(a) and a logical result b_2 from equation 4.9(b). The AND combination of these two signals i.e. $b_3 = b_1$ <u>and</u> b_2 may then be used as a basis for deciding relay operation. A simple pulse width measuring

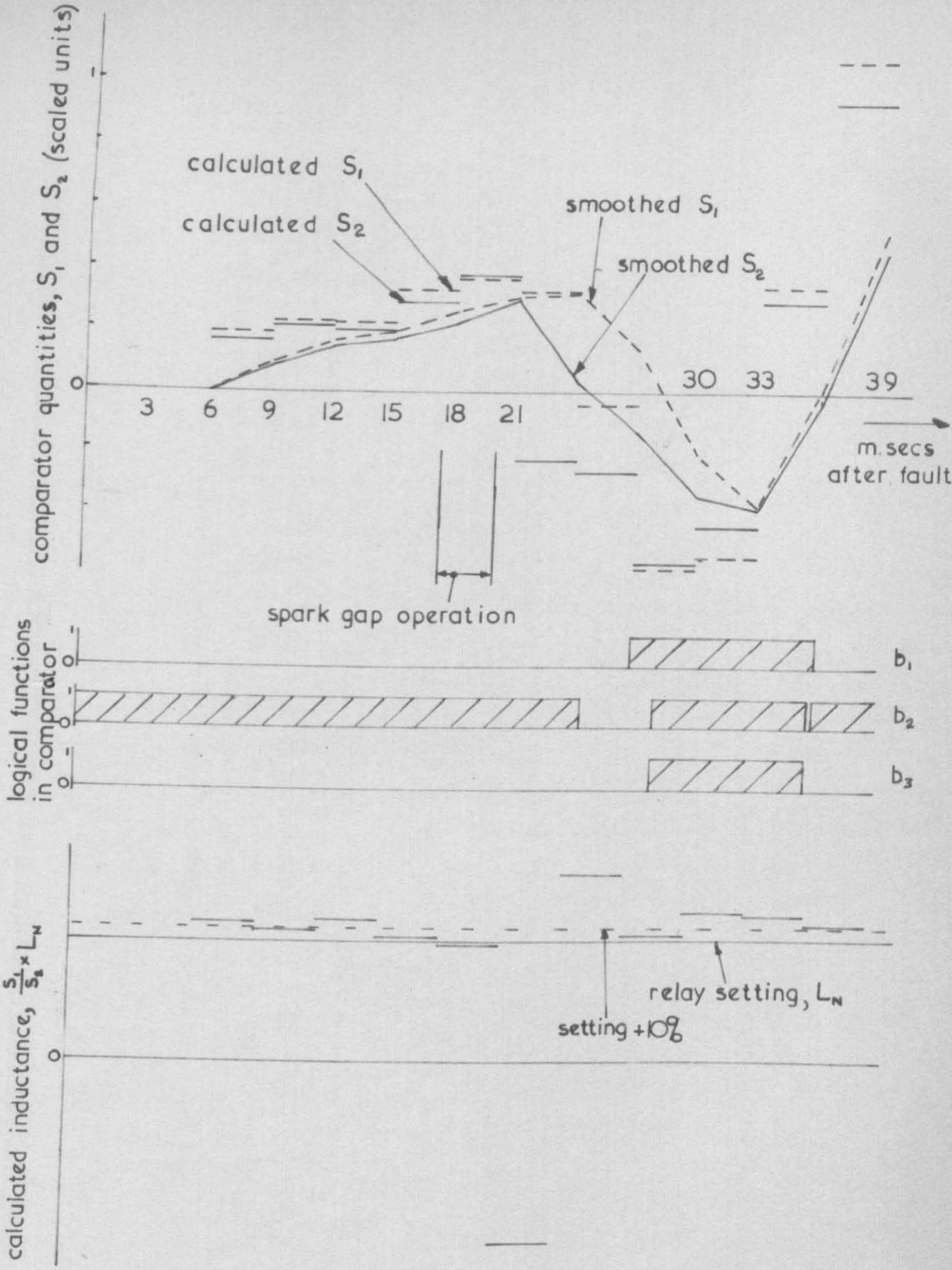


Fig. 4.12 Sampling Relay Performance. Relaying Waveforms Derived from Lumped Parameter Fault Loop with Series Capacitor and Extinguishing Spark Gap—see Table 4.1 for circuit details.
Fault Position, 10% Beyond Relay Setting.

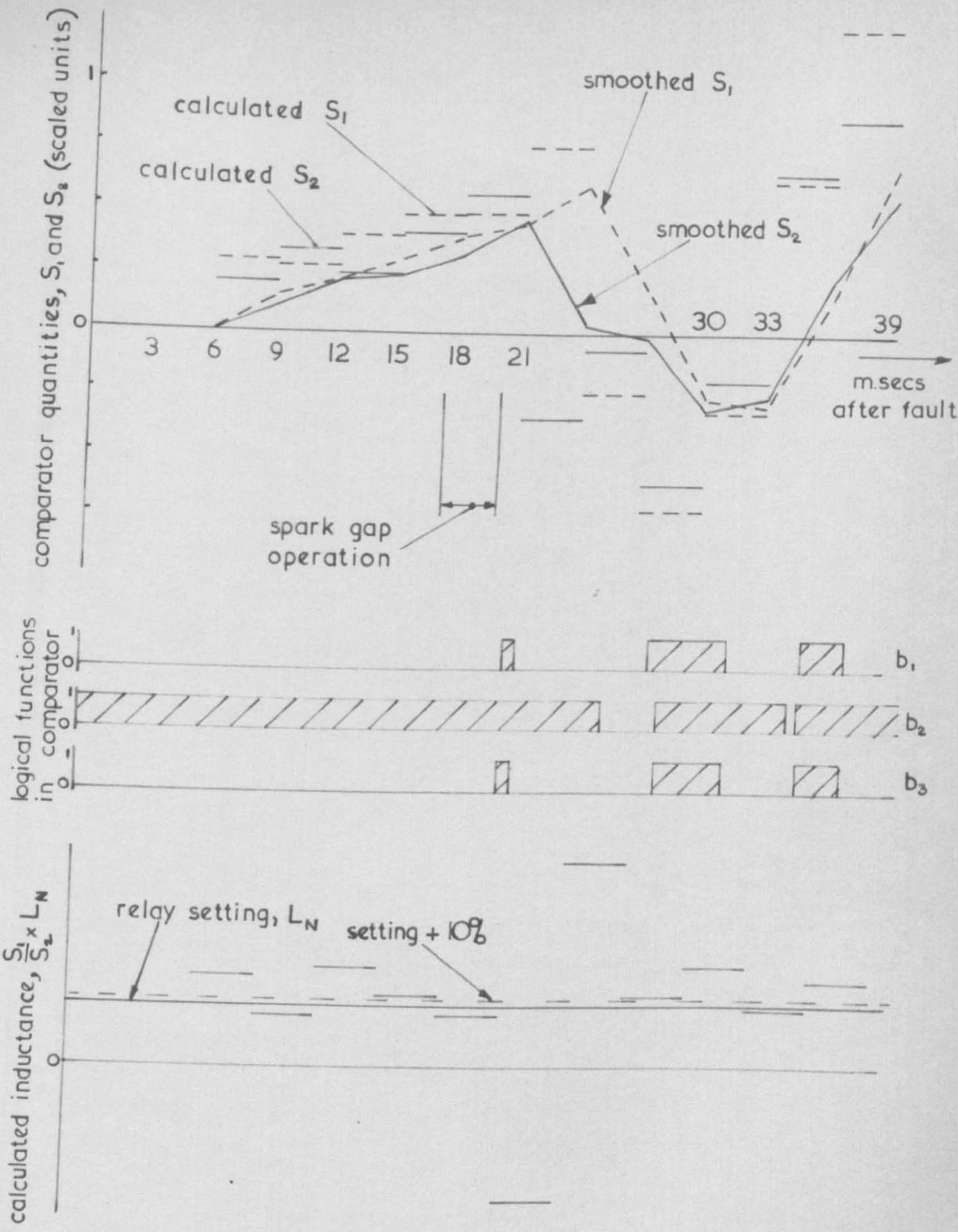


Fig. 4.13 Sampling Relay Performance. Relaying Waveforms Derived from Distributed Parameter Fault Loop with Series Capacitor and Extinguishing Spark Gap — see Table 4.1 for circuit details. Fault Position, 10% Beyond Relay Setting.

TABLE 4.1

ω	=	$100\,\pi$
L_{A1}	=	$33.34/\omega$ H
L_{A2}	=	$0.1/\omega$ H
R	=	0.8515 Ω/Mile
L_1	=	$0.656/\omega$ H/Mile
C_1	=	$5.87/\omega$ μF/Mile
C	=	50% compensation of 230 mile line
l_1 & l_2	=	228 miles
V_S	=	80 v
θ_1	=	$\pi/2$ radians
θ_2	=	$\pi/6$ radians
$\hat{E}_1$	=	100 v
$\hat{E}_2$	=	1 v

Fault loop representation is as shown in Fig. 3.15

circuit would be able to discriminate between a fault within the relay setting and a fault external to the protected line even with protective spark gap operation.

5. REALIZATION OF THE SAMPLING RELAY

The sampling relay depends upon data extracted from small sections of the voltage and current wave-forms supplied by the instrument transformers. The latter must not therefore have significant errors. Current practice with extra high voltage systems is to use bar primary C.T.s. and capacitor voltage division coupled with wound wire V.T.s. This arrangement of capacitor V.T., in particular, can introduce transients which would invalidate measurements made by the sampling relay.

5.1 Choice of Equipment

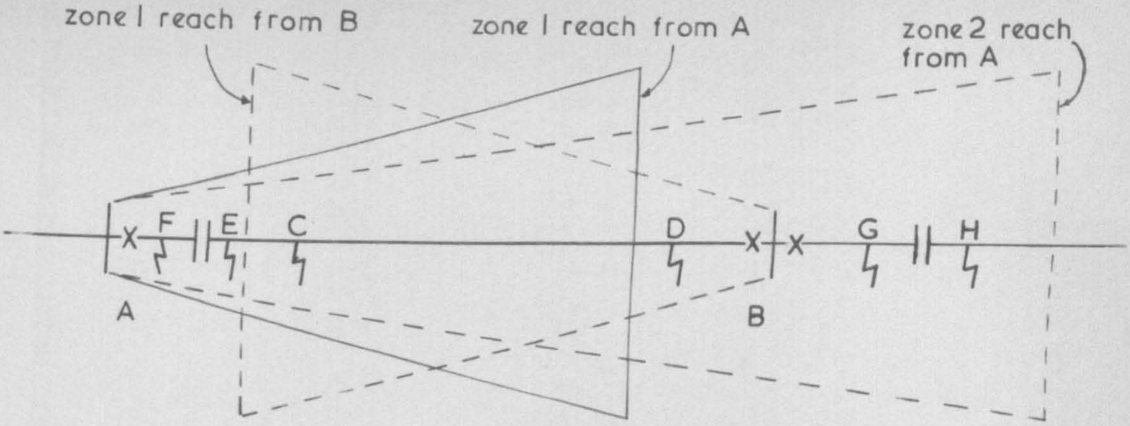
The nature of the signal processing involved is such that the sampling relay must take the form of either a digital computer or alternatively an analogue computer. In comparison with the protective relays used at present, the sampling relay is therefore complicated and expensive, but the choice of protective equipment hardware is influenced mainly by accuracy and reliability and, to a much lesser extent, by the cost. The reliability of analogue circuits is regarded as being acceptable. Digital circuits, on the other hand, are still very much an unknown quantity in the protective equipment industry. Such circuits are used in, for example, frequency relays, but these are not usually required to operate under the arduous conditions associated with a major power system fault. Large and sudden variations in ground potential, for instance, can accompany system faults and digital circuits would need to continue functioning throughout these periods in the very locations where the interference is greatest. The digital computer is faster, more accurate and more easily modified than the analogue computer and once analogue to digital conversion has been achieved all signal processing takes place in a noise free environment. Digital computers and the required communication devices are well developed and can be purchased 'off the shelf'. An entire protective system could thus be composed of well tried and tested equipment and the system could then be programmed to act as a sampling relay. A number of the programming techniques which could be applied have been described in Chapter 4 and, with high speed computers, simultaneous handling of all three phases of a transmission line would be possible. In spite of these superior performance characteristics, the reliability question is unanswered and digital computers are not likely to be used for power system protection for many years.

It was decided, therefore, to develop analogue computer equipment to act as a sampling relay. The fact that a number of analogue computers were available within the University and that suitable digital computers were not so available was also a consideration. The problem of operating analogue computers over a wide range of signal amplitudes is a serious one. As an example, a range of current amplitudes of 30:1 should result in the range of (current)^2 amplitudes being 900:1. The accurate processing of such a range of signals may not be possible with components presently available but with this in mind, the design of the relay was undertaken.

5.2 Principles, Block Diagrams and General Philosophy

The proposal is to equip a section of transmission line, see Fig. 5.1, in such a way that a fault occurring within the line will be detected at each end of the line. The measuring principle to be employed has been described in section 4.3. The relay will function correctly provided that a transient appears in the relaying wave-forms and the sampling relay is, therefore, basically a Zone 1 device. Zone 2 measurement can be made but only during Zone 1 time i.e. about 40 ms. Transfer tripping via a carrier link is required if all internal faults are to be cleared within Zone 1 time. The operation of a complete protective system of a line for all possible fault positions is shown in Fig. 5.1. Zone 3 protection would consist of conventional equipment and is not considered in the figure. The equipment for Zone 1 measurements at one end of a line is shown in Fig. 5.2. The function of the starting relay is to detect the existence of a fault in the system quickly, and to permit the sampling relay to measure for a maximum time of about 40 ms. If a Zone 1 fault is not detected within this time then Zone 2 and Zone 3 relays are left to detect the fault.

The remainder of this chapter is devoted to the design of one of the sampling relays, say the Red-Earth fault relay. The block diagram for the equipment is Fig. 5.3. The operation of the sampling relay depends upon the following derived signals:



(a) PROPOSED SETTING OF SAMPLING RELAY

(b) TABLE SHOWING OPERATION OF PROTECTION FOR FAULTS POSITIONED AS IN (a), ABOVE

			carrier operational		carrier out of order	
fault position	measurement at A	measurement at B	trip time at A	trip time at B	trip time at A	trip time at B
C	zone 1	zone 1	zone 1	zone 1	zone 1	zone 2
D	zone 2	zone 1	zone 1	zone 1	zone 2	zone 1
E	zone 1	zone 2	zone 1	zone 1	zone 1	zone 2
F	zone 1	zone 2	zone 1	zone 1	zone 1	zone 2
G	zone 2	∞	zone 2	zone 2	zone 2	∞
H	zone 2	∞	zone 2	zone 2	zone 2	∞

OPERATING TIMES: zone 1 – 20 – 40 mill. secs.

zone 2 – 1 – 2 secs.

Fig. 5.1 The Proposed Protection Scheme for a Section of Transmission Line

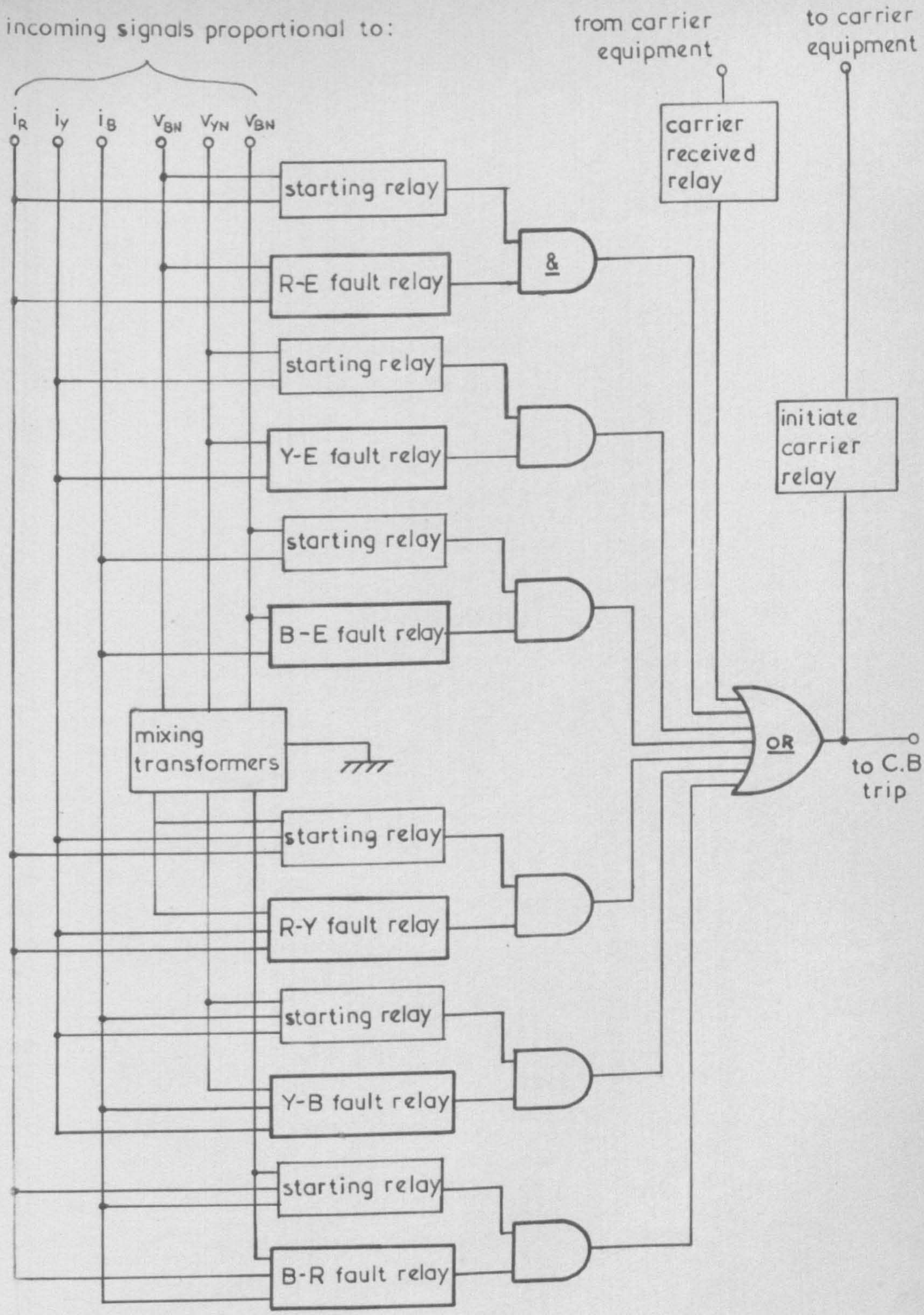
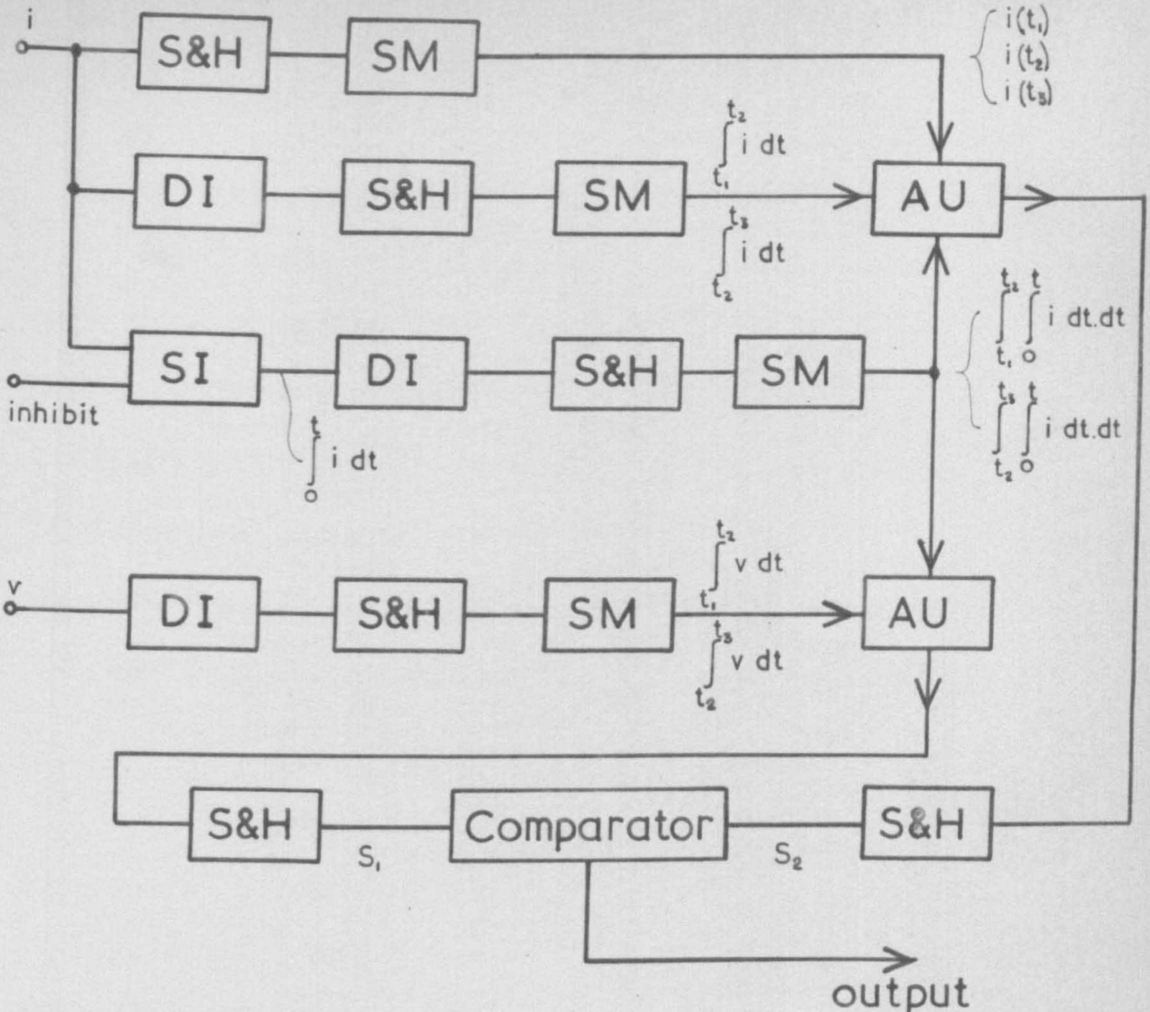


Fig. 5.2 Block Diagram of 3-Phase Zone I Distance Protection Based upon Sampling Relays



LEGEND

i – signal proportional to fault loop current, i_R

v – .. .. voltage, V_{RN}

S&H – sample and hold unit

SM – sample mixing unit

DI – unit forming definite integral

SI – special integration network

AU – arithmetic unit

inhibit – output from starting relay

Fig. 5.3 Block Diagram of Red-Earth Fault Sampling Relay

$i(t_1)$, $i(t_2)$, $i(t_3)$, $\int_{t_1}^{t_2} idt$, $\int_{t_2}^{t_3} idt$, $\int_{t_1}^{t_2}\int_0^t idtdt$,

$\int_{t_2}^{t_3}\int_0^t idtdt$, $\int_{t_1}^{t_2} vdt$ and $\int_{t_2}^{t_3} vdt$; i and v are proportional to the relaying quantities; t_1, t_2, and t_3, are consecutive instants in time. These signals are mixed together in an arithmetic unit i.e. a network of summing amplifiers and multipliers, in order to form the two comparator signals S_1 and S_2 according to equation 4.8. The actual value of i is sampled by three Sample and Hold units at times

$(t'_1, t'_4, t'_7, t'_{10},)$, $(t'_2, t'_5, t'_8, t'_{11} \ldots)$ and

$(t'_3, t'_6, t'_9, t'_{12} \ldots)$ where $t_2 - t_1 = t_3 - t_2 = t'_2 - t'_1 = t'_3 - t'_2 = t'_4 - t'_3$ etc. and the values of $i(t_1)$, $i(t_2)$ and $i(t_3)$ are related to these samples at times t'_1, t'_2, t'_3, t'_4 ... as follows :-

If $t_1 = t'_1$, $t_2 = t'_2$ & $t_3=t'_3$ reckoned at time t, $t'_3<t<t'_4$,
then $t_1 = t'_2$, $t_2 = t'_3$ & $t_3 = t'_4$ reckoned at time t, $t'_4<t<t'_5$
and $t_1 = t'_3$, $t_2 = t'_4$ & $t_3 = t'_5$ reckoned at time t, $t'_5<t<t'_6$.

This principle of mixing the outputs of three Sample and Hold units means that if n samples are obtained and three consecutive samples are required for a calculation then n - 2 different calculations can be performed with a minimum amount of equipment. The wave-forms of $i(t_1)$, $i(t_2)$ and $i(t_3)$ obtained in such a way are illustrated in Fig. 5.4. This process of mixing samples is

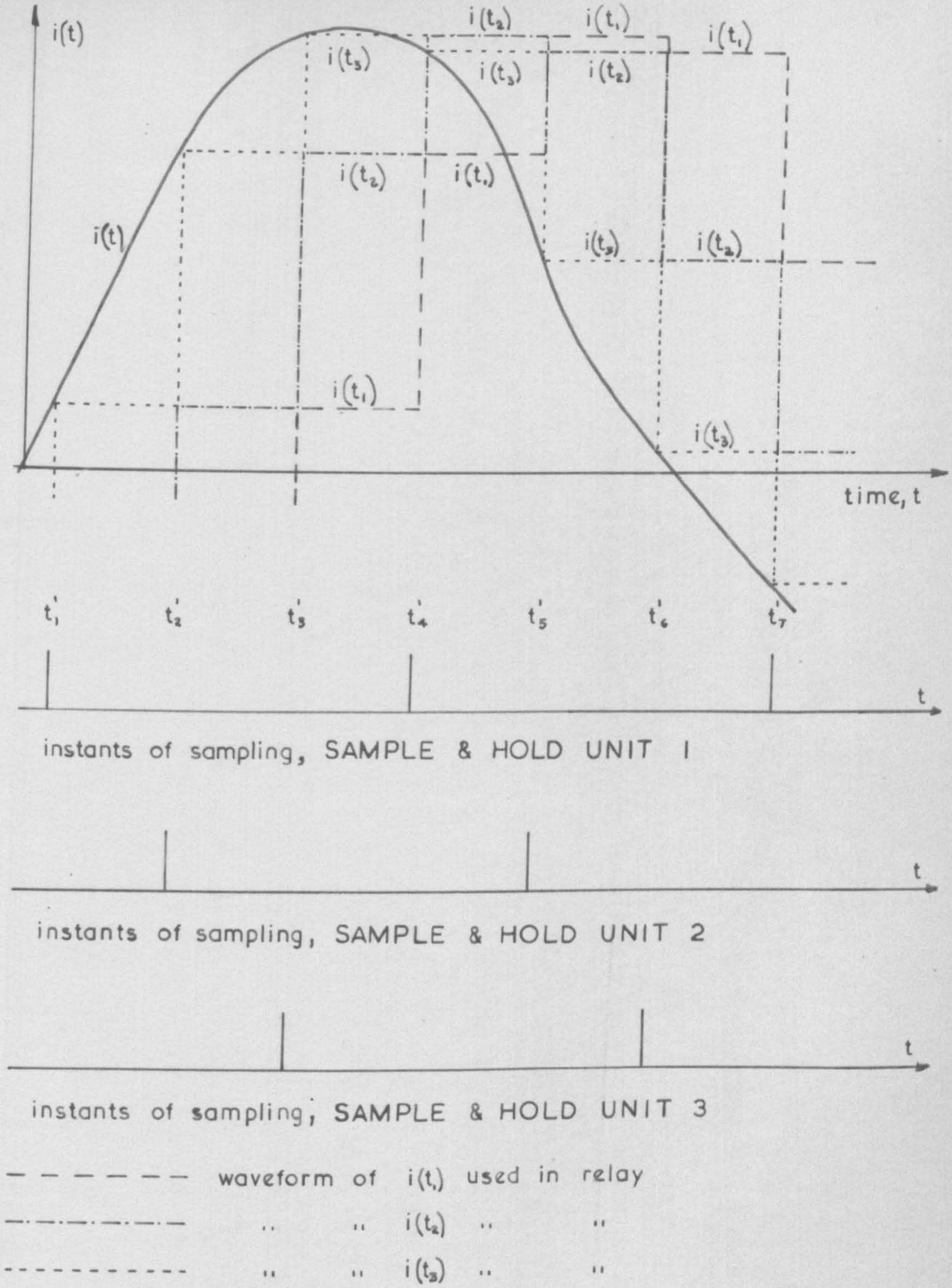


Fig. 5.4 Organisation of Waveform Sampling. Instantaneous Sampling

clearly applicable to other signals e.g. if

$\int_{t'_2}^{t'_3} idt$ has been obtained and is substituted for $\int_{t_2}^{t_3} idt$ for the period $t'_3 \rightarrow t'_4$ then it can be substituted for $\int_{t_1}^{t_2} idt$

for the period $t'_4 \rightarrow t'_5$.

The detailed circuitry associated with each of the blocks shown in Fig. 5.3 is described in section 5.3. The supervisory control circuit which is obviously necessary to co-ordinate and control the harmonious operation of the many separate circuits, was designed to produce the signals shown in Fig. 5.5a. The circuit must provide pulses which can be directed to initiate particular operations; for instance,a pulse lasting for the period $t'_1 \rightarrow t'_2$ may be used to switch on an integration network so that the signal $\int_{t'_1}^{t} idt$ emerges for $t'_1 < t < t'_2$. The basic logical function u in Fig. 5.5 is obtained from a free running oscillator. The period of u is the time $t_2 - t_1 = t_3 - t_2$, so that if this interval is chosen to be 2.5 ms. then the frequency of u must be 400 Hz. A slight drift in the frequency does not affect the accuracy of the equipment. The other main function A, B and v are derived from u via bistable circuits. The remaining functions are then obtained using plain logical gates. This is conveniently achieved using boolean algebra. The boolean expressions of Fig. 5.5(b) are convertible directly to the NAND gate circuits of Fig. 5.6 which shows the whole of the supervisory control circuit. The circuit output signals

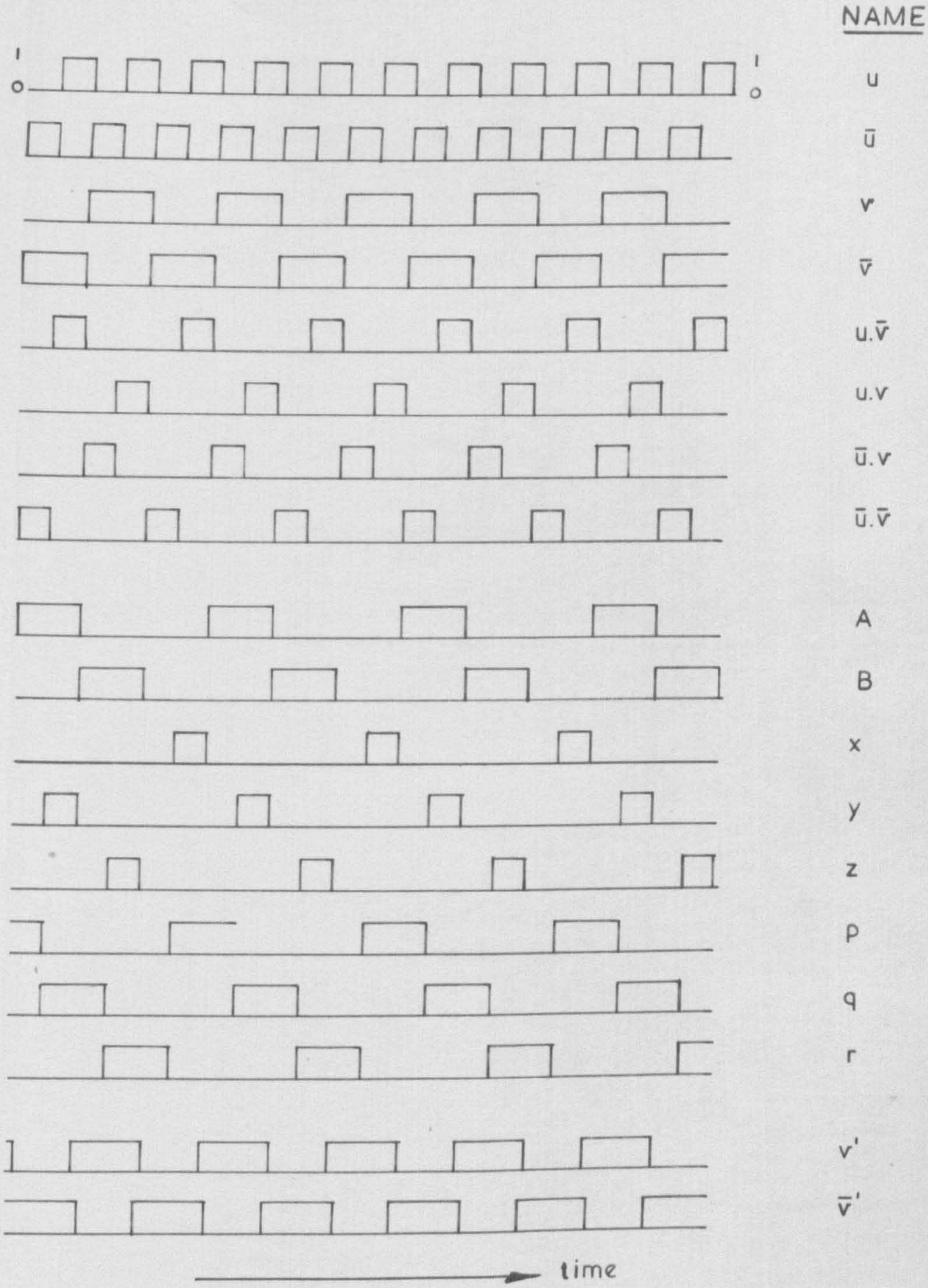


Fig. 5.5(a) Supervisory Control Signals — Pictorial Relationships.

NAME	DERIVATION (Boolean NAND form)	DESTINATION OF LOGICAL SIGNAL
u	oscillator	driving of logic circuits
$\bar{u}$	$\bar{u}$	used in comparator
v	u thro' bistable	control of $I_1 = \int_{t_1'}^{t_2'} f(t)\, dt$
$\bar{v}$	$\bar{v}$	control of $I_2 = \int_{t_2'}^{t_3'} f(t)\, dt$
$u.\bar{v}$	$\overline{\overline{u.\bar{v}}}$	control of sampling of I_1
$u.v$	$\overline{\overline{u.v}}$	control of sampling of I_2
$\bar{u}.v$	$\overline{\overline{\bar{u}.v}}$	used in the mixing of the samples of I_1
$\bar{u}.\bar{v}$	$\overline{\overline{\bar{u}.\bar{v}}}$	& I_2 to form $\int_{t_1}^{t_2} f(t)\, dt$ & $\int_{t_2}^{t_3} f(t)\, dt$
A	u thro' counter	driving of logic circuits
B	" "	" " "
x	$\overline{\overline{u.\bar{A}.\bar{B}}}$	sampling control of $i_1(t) = i(t_1'), i(t_4') \ldots$
y	$\overline{\overline{A.u}}$	" " $i_2(t) = i(t_2'), i(t_5') \ldots$
z	$\overline{\overline{B.u}}$	" " $i_3(t) = i(t_3'), i(t_6') \ldots$
p	$\overline{\overline{x.\bar{y}}.\overline{A.\bar{y}}}$	used in sample mixing to form
q	$\overline{\overline{y.\bar{z}}.\overline{B.\bar{z}}}$	$i(t_1)$, $i(t_2)$ & $i(t_3)$ from
r	$\overline{\overline{p.q}}$	$i_1(t)$, $i_2(t)$ & $i_3(t)$, see above
v'	v thro' pulse stretcher	used instead of v to form I_1
$\bar{v}'$	$\bar{v}$ " " "	" " $\bar{v}$ " I_2

Fig. 5.5(b) Supervisory Control Signals — Table Showing Derivation (in Boolean NAND Form) and Eventual Destination.

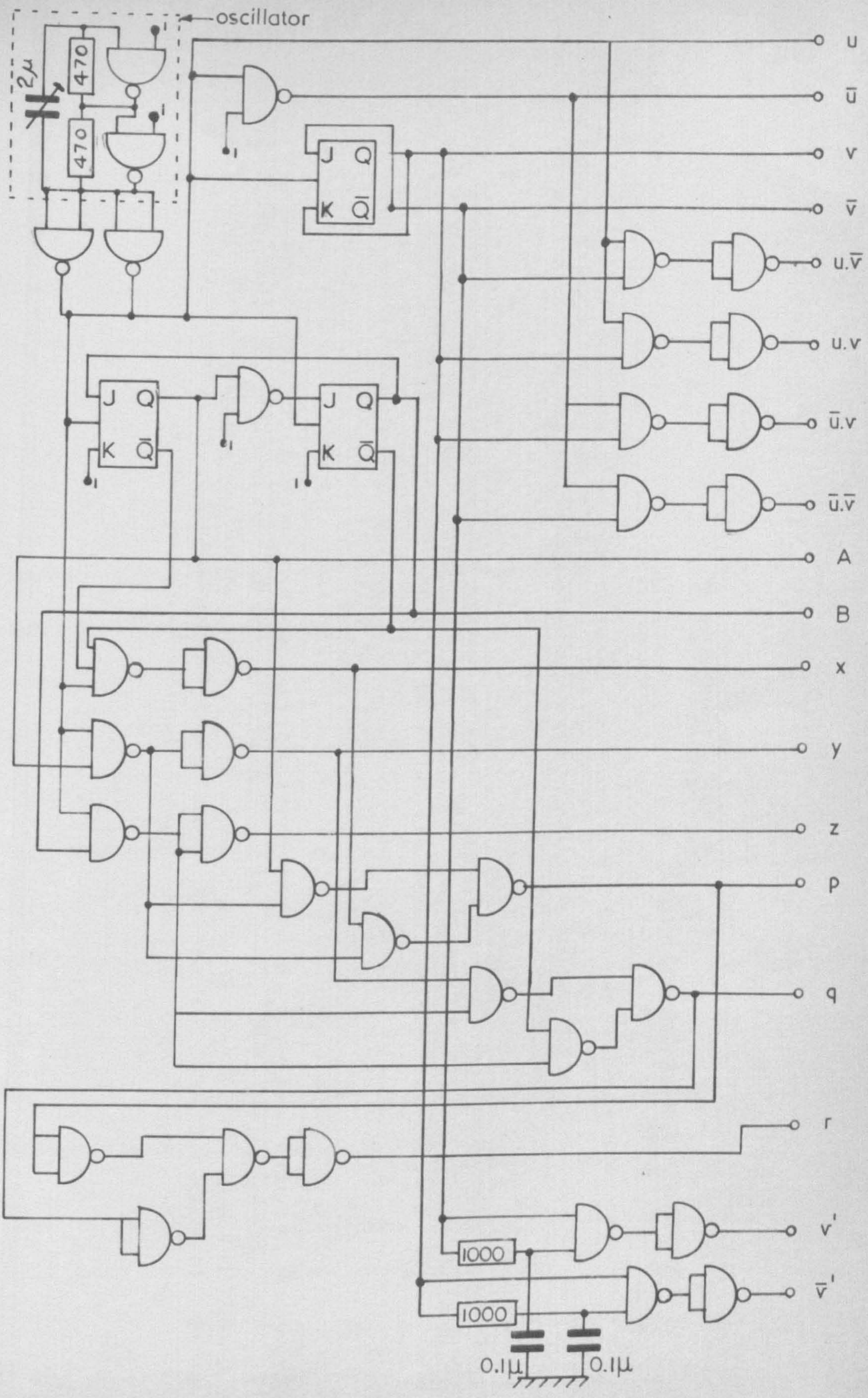


Fig. 5.6 Generation of Supervisory Control Signals

are displayed in Fig. 5.7 for comparison with Fig. 5.5a. The logical levels obtained from the integrated circuits used are 0-0.8 volts (a logical 0) and 2-5 volts (a logical '1'). Since field effect transistors (FET's) were used extensively in the analogue circuits and these require gating signals changing between -15 volts and +15 volts, driving circuits were required in order to obtain these much larger logical levels. The driving circuit design is shown in Fig. 5.8.

All the circuits, with the exception of the comparator, summing amplifiers and multipliers which are part of the analogue computers, were assembled on to 27-2.5 x 4.75 inch copper-back Veroboards. This layout was guided by a desire to maintain flexibility of design and interchangeability of circuit boards rather than minimum number of components. Many of the circuit boards were identical and were grouped as follows :

5 circuit boards containing all the supervisory control circuits.

6 identical circuit boards each containing 2 Sample and Hold circuits.

6 identical circuit boards each containing an integration network.

7 identical circuit boards each containing 3 analogue switches.

3 circuit boards associated with the formation of the integral $\int_0^t i\,dt$.

Three types of transistor and four types of integrated circuit were used and total component costs were less than £80. This figure was made up mainly of Veroboards, plugs and connecting pins and would be considerably reduced in an optimised design.

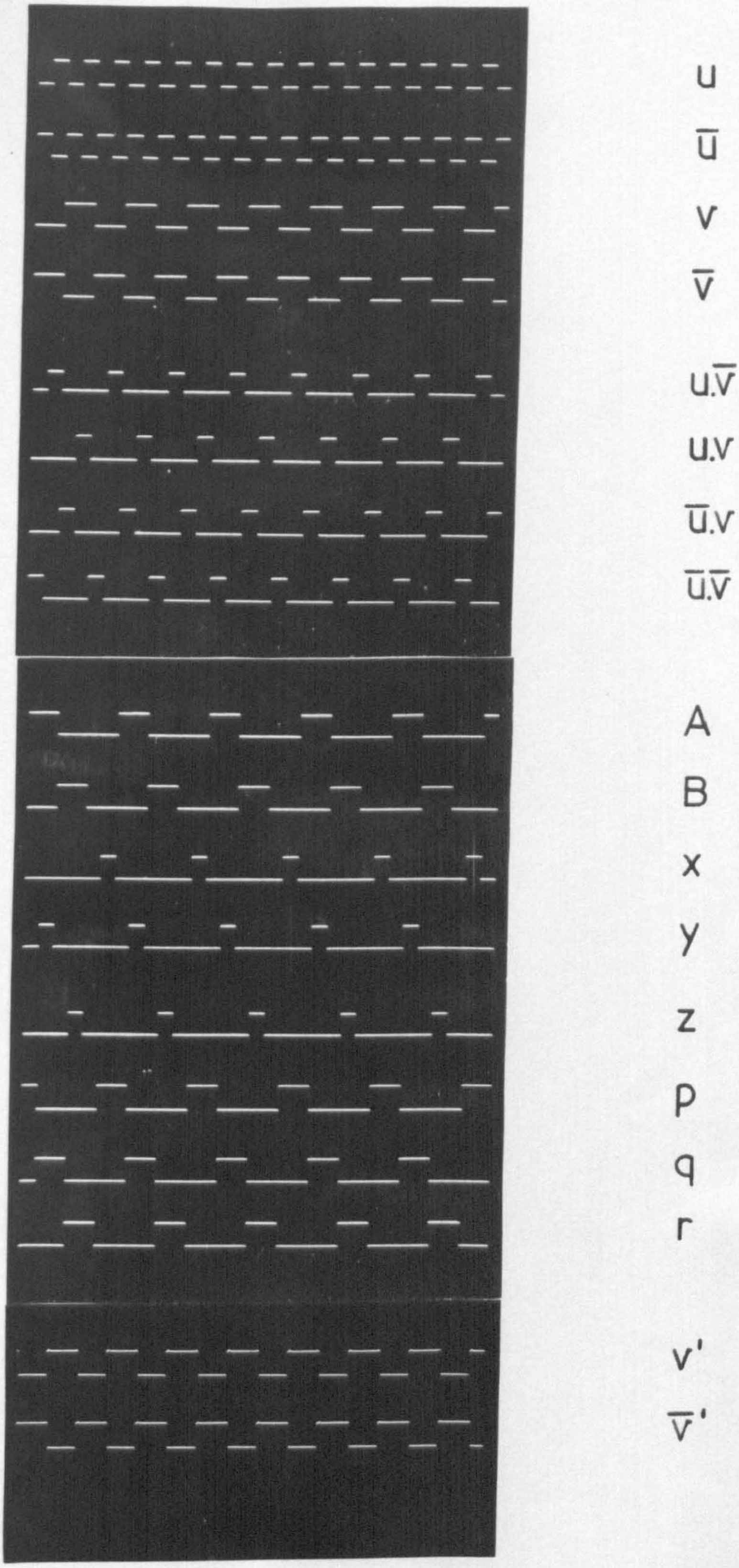


Fig. 5.7 Supervisory Control Circuit Output for Comparison with Fig. 5.5(a)

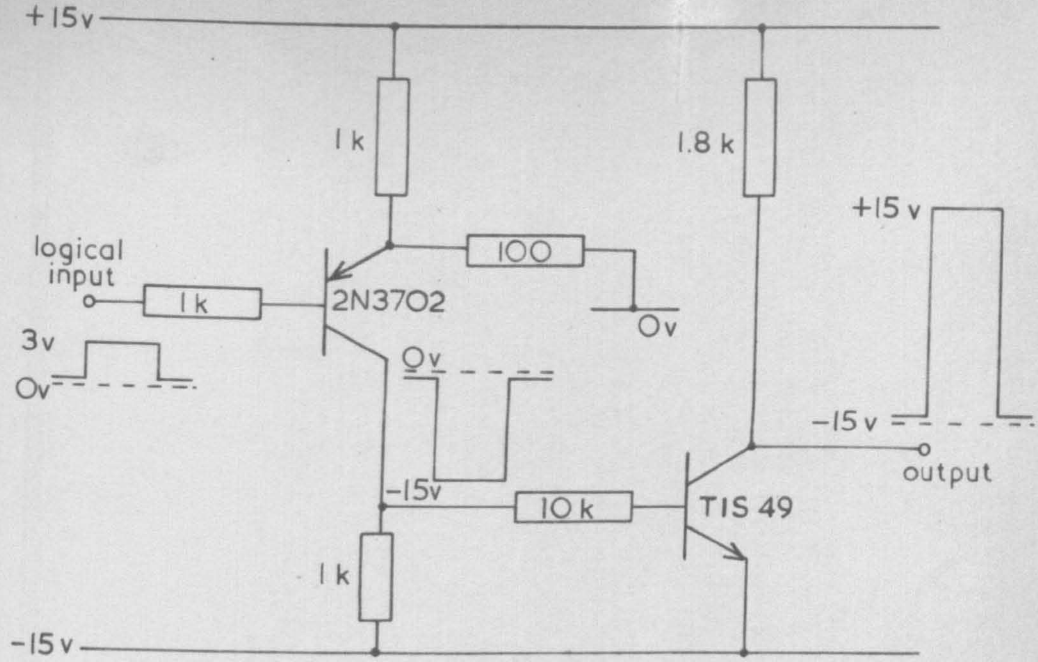


Fig. 5.8 Driving Circuit for F.E.T.'s

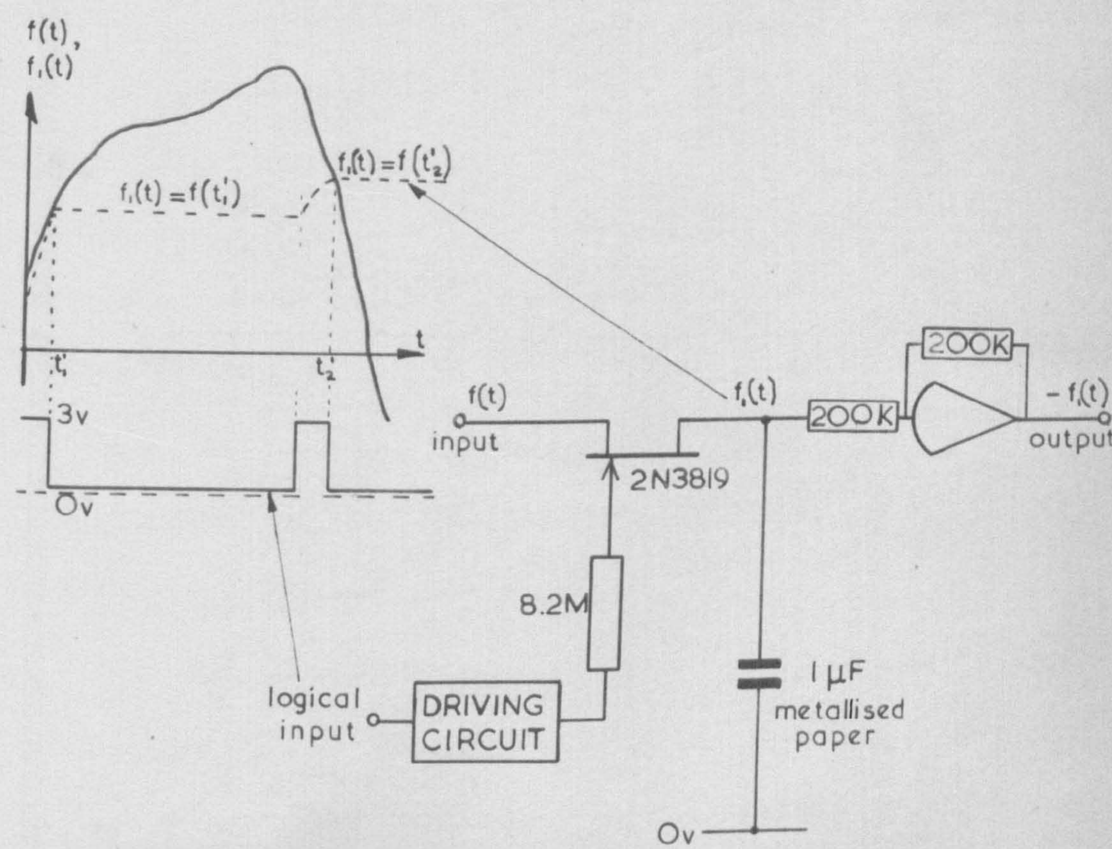


Fig. 5.9 Sample & Hold Circuit

The 41 chopper stabilised operational amplifiers which were part of the analogue computers were used for convenience. Using modern integrated circuits these would have been constructed for £1-50 each plus the cost of the precision input and feedback resistors required. Lastly, the 4 quarter square multipliers which were also part of the analogue computers could have been replaced by modern potted units at £60 each.

5.3 The Circuits

The design and operation of the Sample and Hold circuit used is shown in Fig. 5.9. A high speed, high accuracy design could have been used but, as explained in section 4.3.2, such equipment can cause large errors. The time constant of the circuit during sampling is governed by the signal source resistance, FET_{ON} resistance and the value of the sampling capacitor. A slight degradation in the 'held' sample due to the finite load impedance was experienced. This was compensated for, after the sample mixing stage.

The analogue switch circuit, shown in Fig. 5.10, formed a fundamental building unit and was used in the construction of the sample mixing circuits.

The operation and circuit of an integrator, used in conjunction with an analogue computer amplifier, is illustrated in Fig. 5.11. With both FET's OFF, the circuit operates like a conventional active integrator. When FET_1 is switched ON, the integrator capacitor is discharged but since the discharge

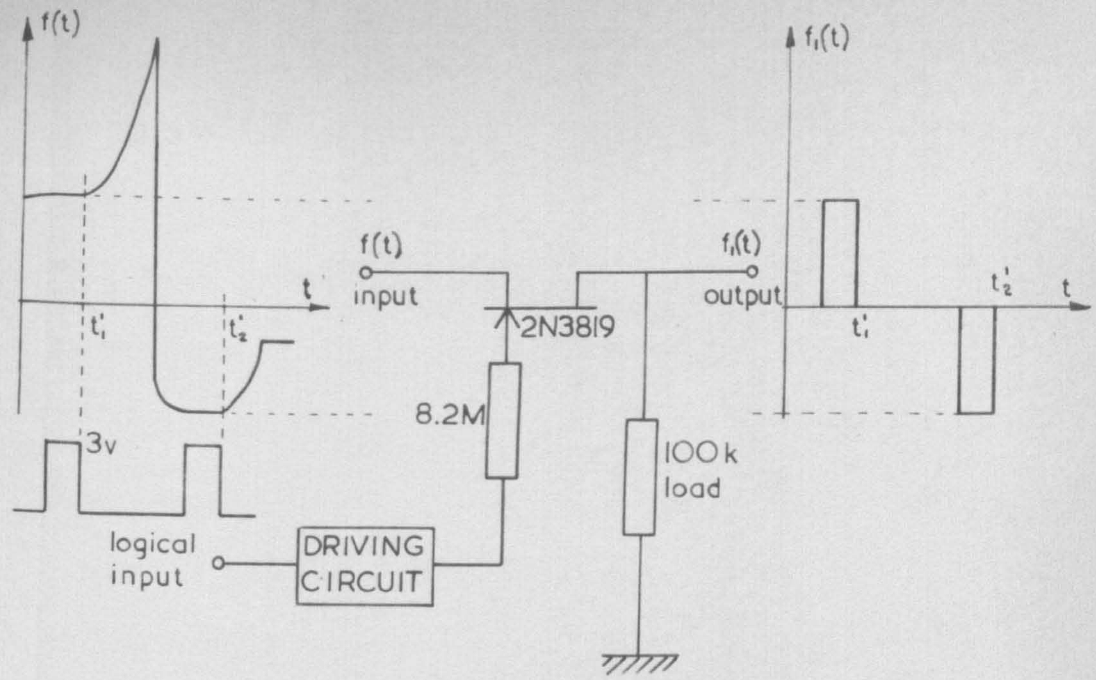


Fig. 5.10 Analogue Switch Circuit

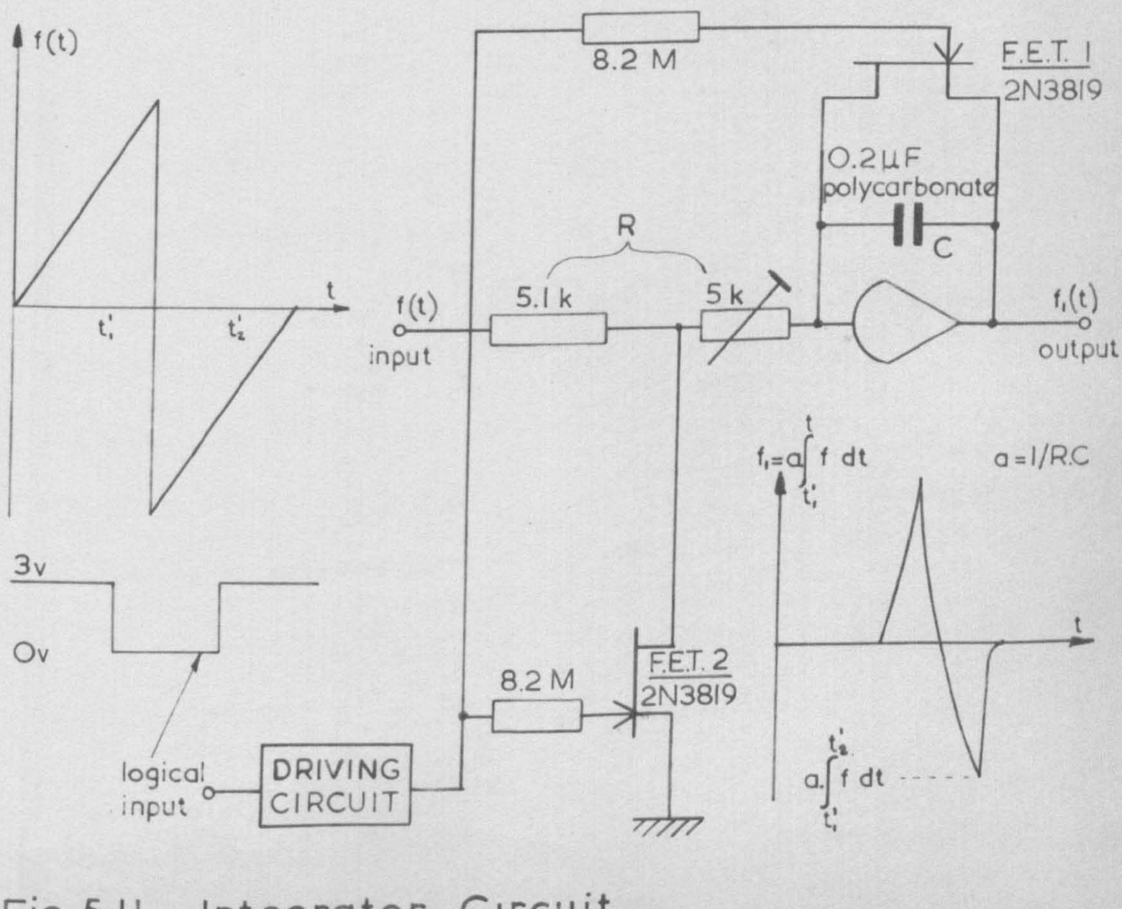


Fig. 5.11 Integrator Circuit

is not a short circuit the negative feedback on the amplifier is incomplete. The input signal is virtually removed from the amplifier by FET_2 which is switched ON at the same instant as FET_1 thus reducing the residual output signal to a negligible proportion.

The formation of the signal which represents $\int_0^t idt$ requires that the time t = 0 is known. Since the protective scheme is based upon this time corresponding to zero charge on the series capacitor in the line, the following conditions for starting off the integration were formulated.

1. The relaying current is in the steady state i.e. no transient is present.

2. The relaying current is at a peak.

3. The time elapsed since the last setting of t = 0 is greater than an arbitrary time, say 60 seconds. The integration must be reset periodically because noise and drift will always be present in an integration process and this reveals itself as a slow change in the integration output signal and eventually, saturation of the amplifier. The steady state condition is determined by the starting relay and an inhibit signal is sent in the event of a transient being detected. The relaying current peak was detected by phase lagging the i(t) waveform, followed by detection of zeros and eventual production of a logical signal, the trailing edge of which coincides with the peak of the fundamental component of the current. The method

does not take proper account of strong harmonies which may be present but since such distortion is not expected and since fault current will most usually be larger than load current, the error involved is very small. The 60 second timer may take a variety of forms. In the experimental work a very simple 2 second timer was used which relied upon the charging of a capacitor coupled with the operation of a clocked JK bistable integrated circuit. The complete circuit for the formation of $\int_0^t idt$ is shown in Fig. 5.12 and a sample of the wave-forms obtained from it are shown in Fig. 5.13. Ideally, the integrator output will be close to zero when the resetting pulse appears at the integrator. The minimum reset time depends upon the accuracy of resetting required and the ability of the discharge circuit to cope with the charge residing on the integration capacitor. More powerful FET's can be used or less powerful devices connected in parallel can improve the reset time. In the experimental work a reset time of 0.5 ms. was easily achieved and was considered to be short enough for the application.

The complete circuit of the sampling relay was connected as shown in Fig. 5.14. The comparator used included some equipment to smooth the incident wave-forms, a summing amplifier and an electro-mechanical relay arranged to trip for the condition $S_2 - S_1 > 0$. The inertia of the moving parts of the relay and the inductance of its coil were effective, to some extent, in discriminating between meaningful information based upon the ratio S_2/S_1 and the effect of spurious irregular-

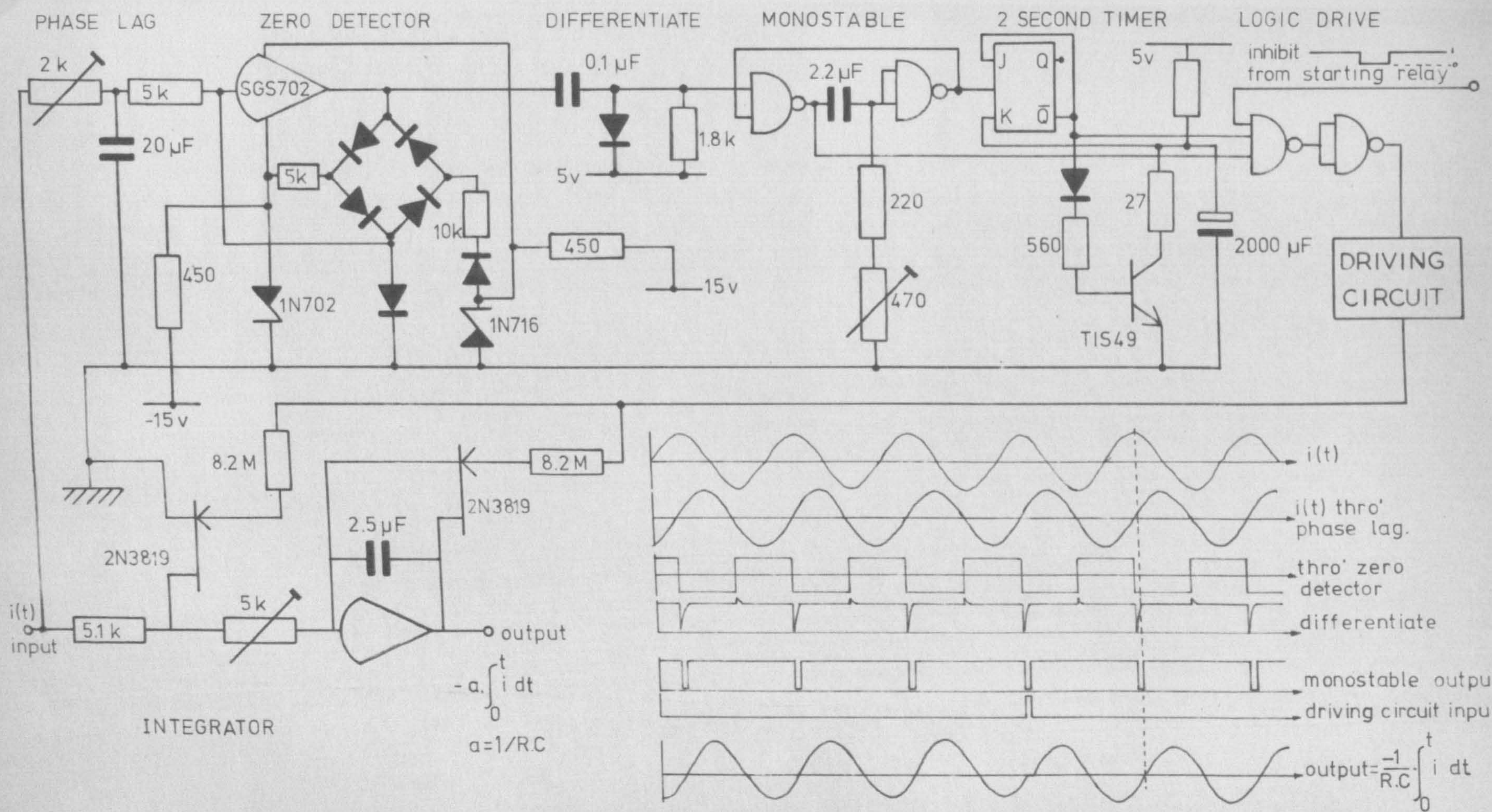


Fig. 5.12 Formation of $a.\int_0^t i\,dt$ Including Resetting Circuits

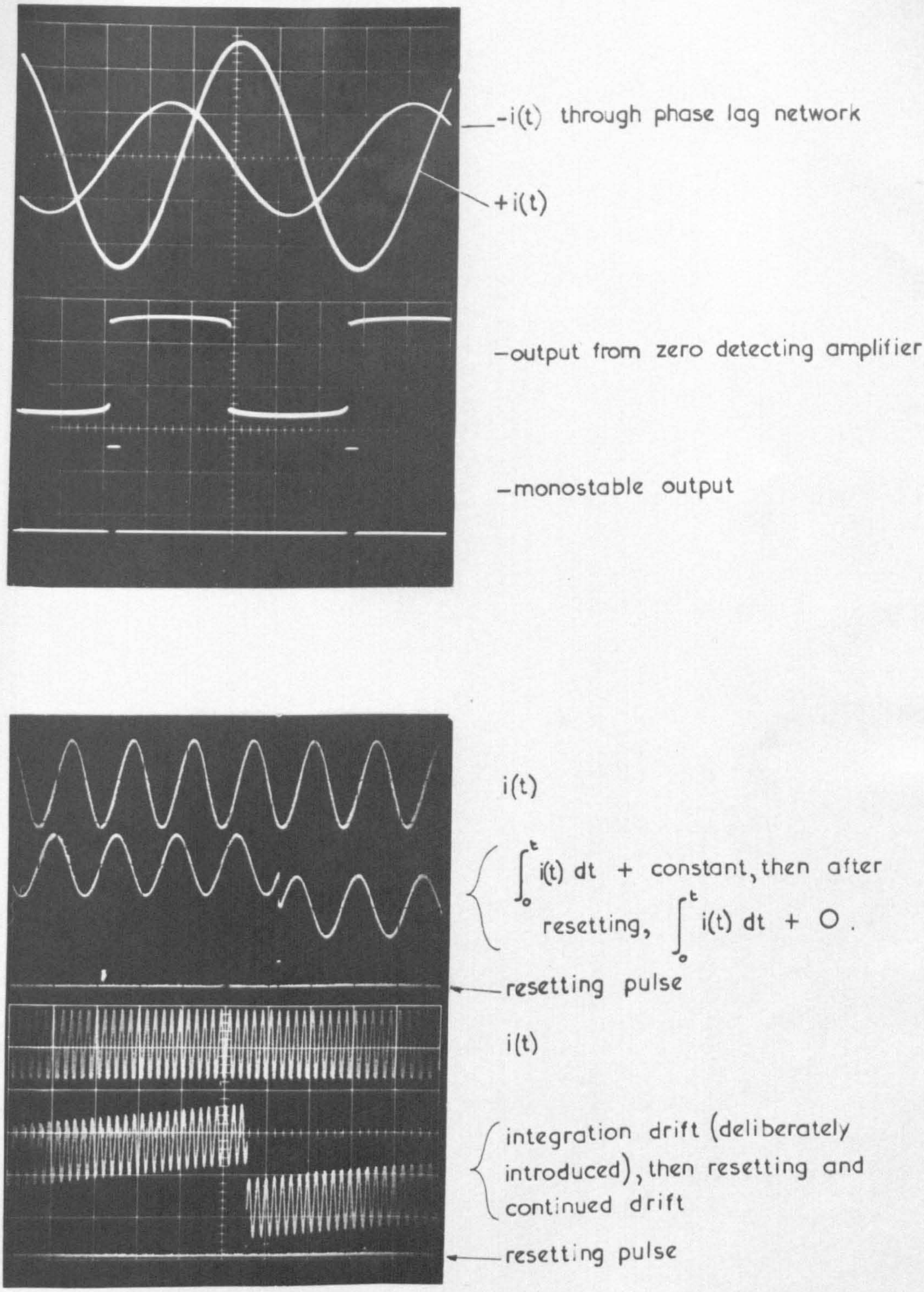


Fig. 5.13 Waveforms Obtained in the Derivation of $\int_0^t i\,dt$

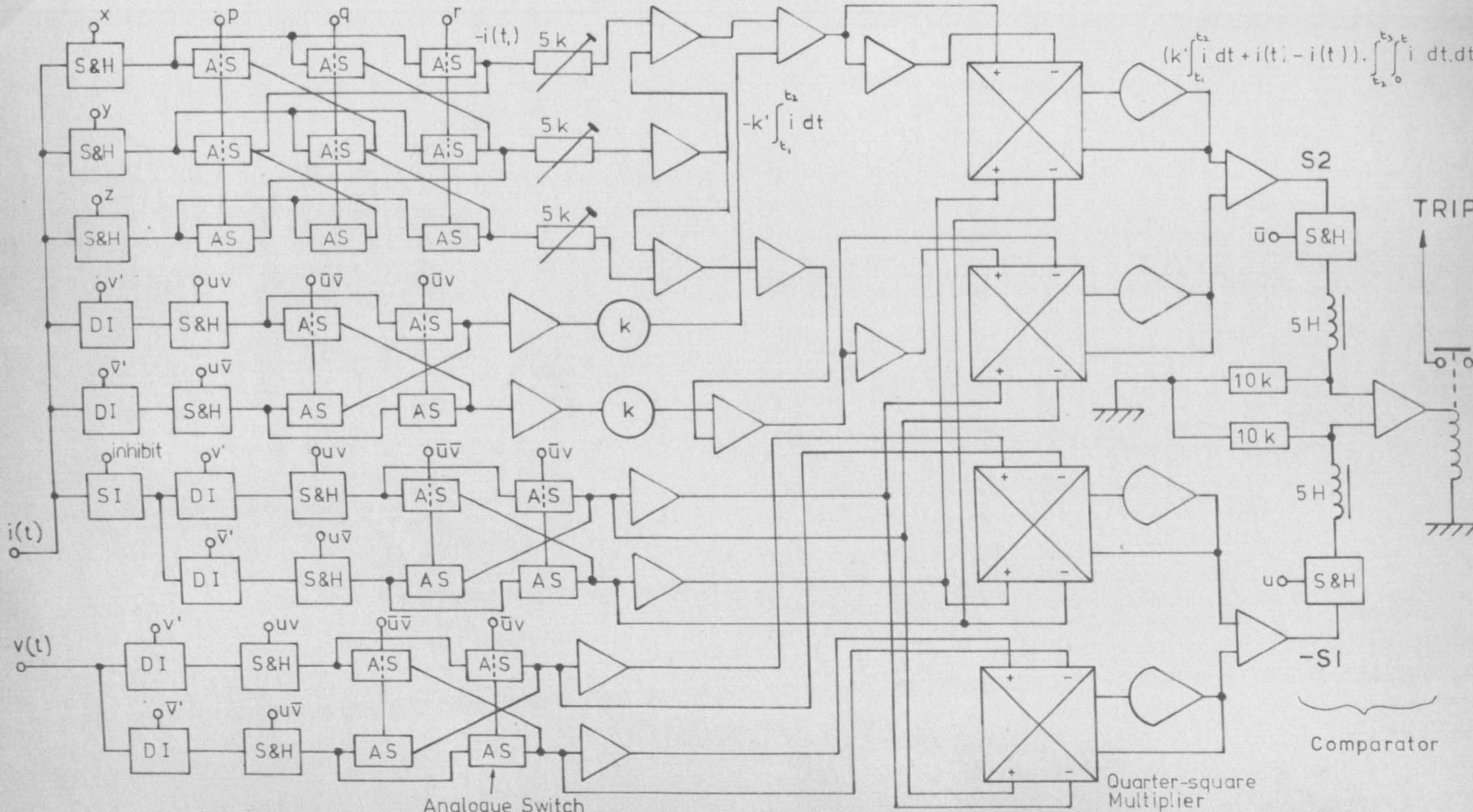


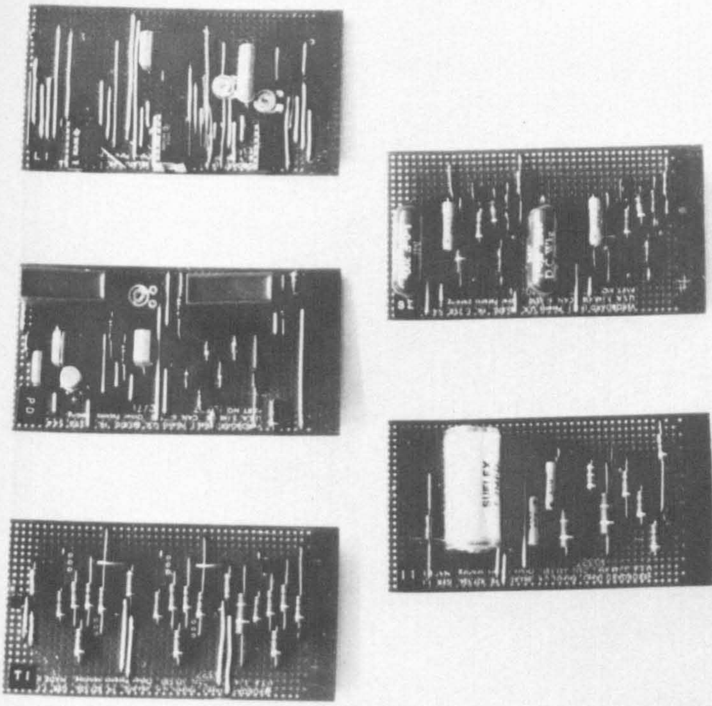
Fig. 5.14 Sampling Relay Circuit Connections Also Showing Control Signals

ities in the S_1 and S_2 wave-forms caused by imperfections in the circuit performance. The setting of the relay could be varied in a number of ways, although it proved to be most convenient to vary the feedback resistors associated with the final stage amplifiers used in the derivation of S_1 and S_2. Photographs of the sampling relay equipment are shown in Figs. 5.15 to 5.18. If integrated circuit operational amplifiers had been used exclusively and if the circuits had been designed to use the minimum number of components, a considerable economy of volume as well as of cost would have resulted.

5.4 Testing and Relay Characteristics

Although the relay is not able to measure accurately in the steady state, it proved to be convenient to check it and undertake preliminary tests with steady state current and voltage signals. These were obtained from an R-L circuit connected to the a.c. mains via variable ratio and isolation transformers. The frequency of the supervisory circuit oscillator was adjusted so that the processes involved in the relay were repeated every cycle of the a.c. voltage, thus enabling wave-forms to be examined closely without resorting to the use of a storage oscilloscope or other equipment.

In the calibration of the Sample and Hold and Integration circuits it was possible to check signal validity easily, but as the signals progressed through the arithmetic unit they became mixed with other signals and the resultant combinations bore



Circuits to produce some of the Supervisory Control Signals

Two Sample & Hold Circuits

Phase Lag & Zero Detector used in Resetting Circuit

Circuit to form Definite Integrals

Three Analogue Switches

Fig.5.15 Selection of Circuits used in Sampling Relay

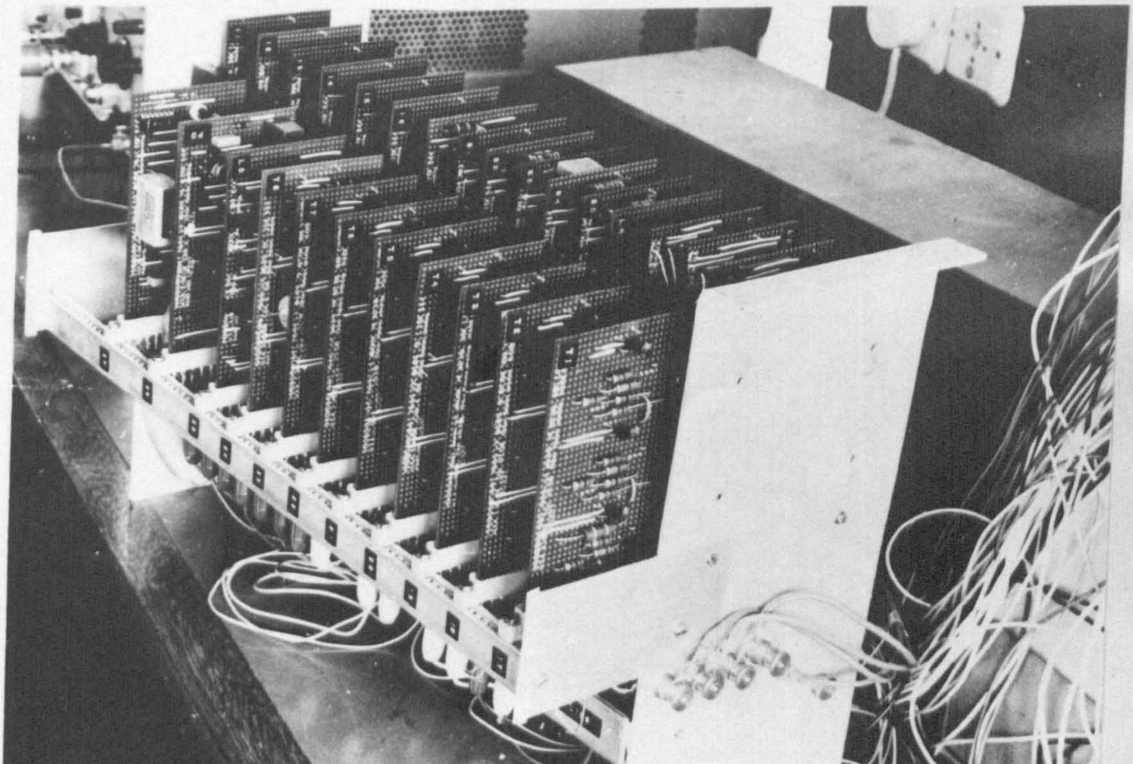


Fig. 5.16 Assembly of Circuit Modules Used

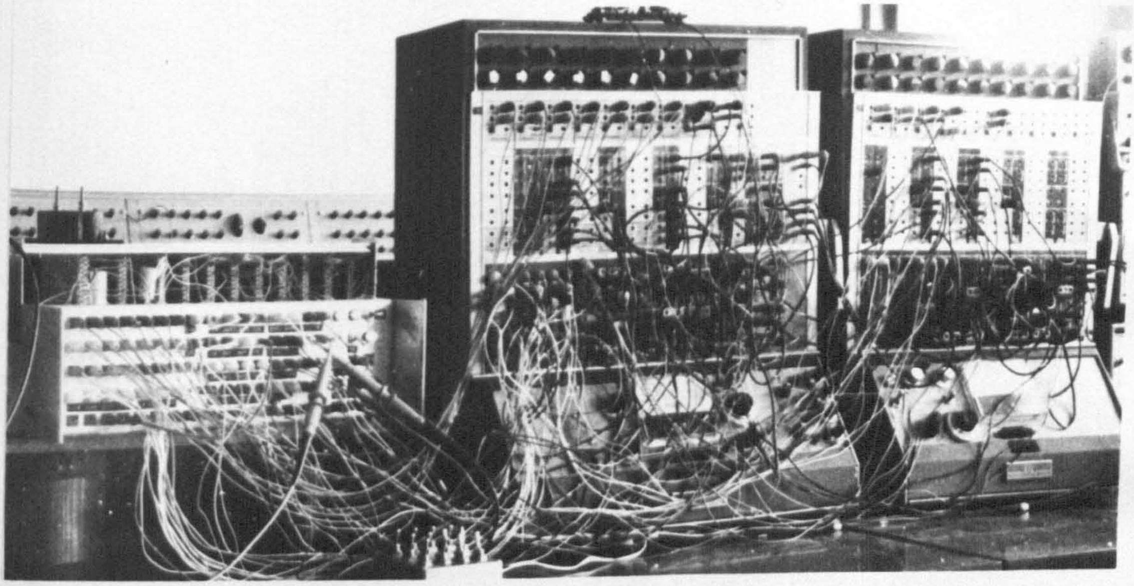


Fig. 5.17 Overall View of Sampling Relay Showing Analogue Computers

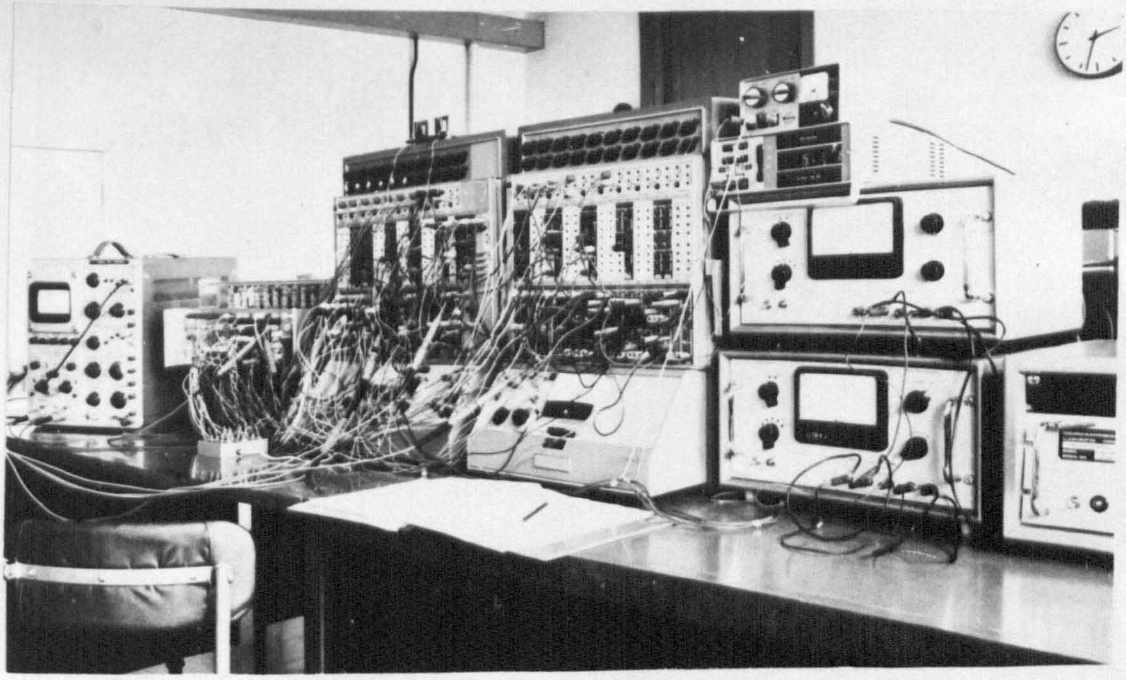
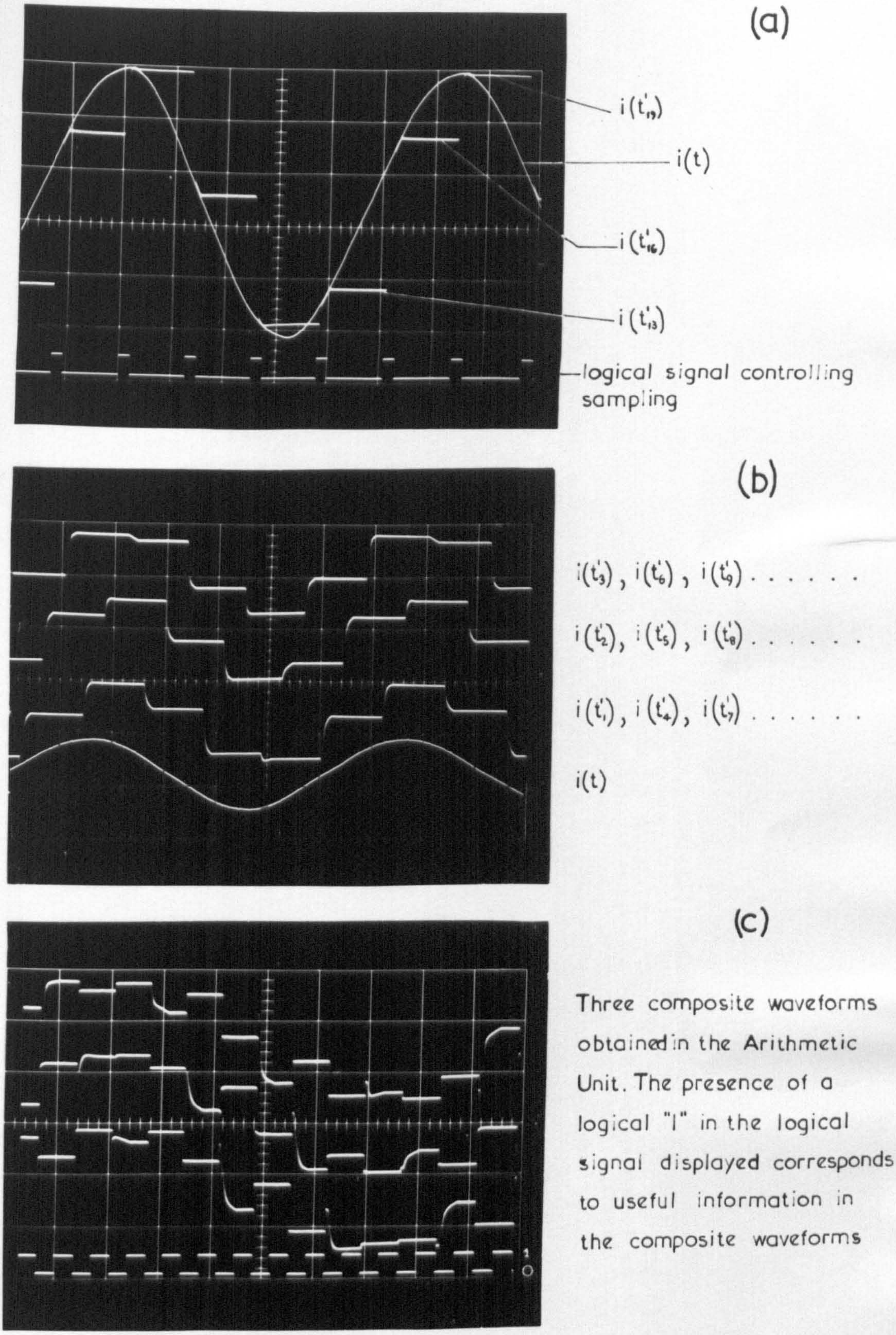


Fig. 5.18 Sampling Relay with Associated Test Equipment

little resemblance to their component wave-forms. For this later stage of testing, signals predicted by a digital computer were used as a standard for comparison with those produced by the relay. A Teletype terminal operating on a COTAN System and linked to a remote KDF9 digital computer was made available adjacent to the equipment and this enabled interactive use of the digital computer for quick checking of wave-forms appearing within the sampling relay. The computer program used was based upon the equations and techniques described in sections 3.3.1 and 4.3.3 and was written for translation by a fast compiler which resulted in typical run times of 3 seconds being achieved.

During the steady state development and testing, photographs of some of the main wave-forms occurring within the relay were taken. The performance of the Sample and Hold circuits is shown in Fig. 5.19(a) and (b) and three composite wave-forms obtained from within the arithmetic unit are shown in Fig. 5.19(c). A supervisory control signal is always available for comparison with the processed signals and in Fig. 5.19(c) the logical '1's (the uppermost parts of the square wave) in the control signal correspond to useful information in the three other signals. At any stage this logic can be applied to an analogue switch, for example to eliminate redundant parts of a wave-form. In Fig. 5.20, the derivation of two of the definite integral signals is shown.



Three composite waveforms obtained in the Arithmetic Unit. The presence of a logical "1" in the logical signal displayed corresponds to useful information in the composite waveforms

Fig. 5.19 Composition & Use of Sampled Signals

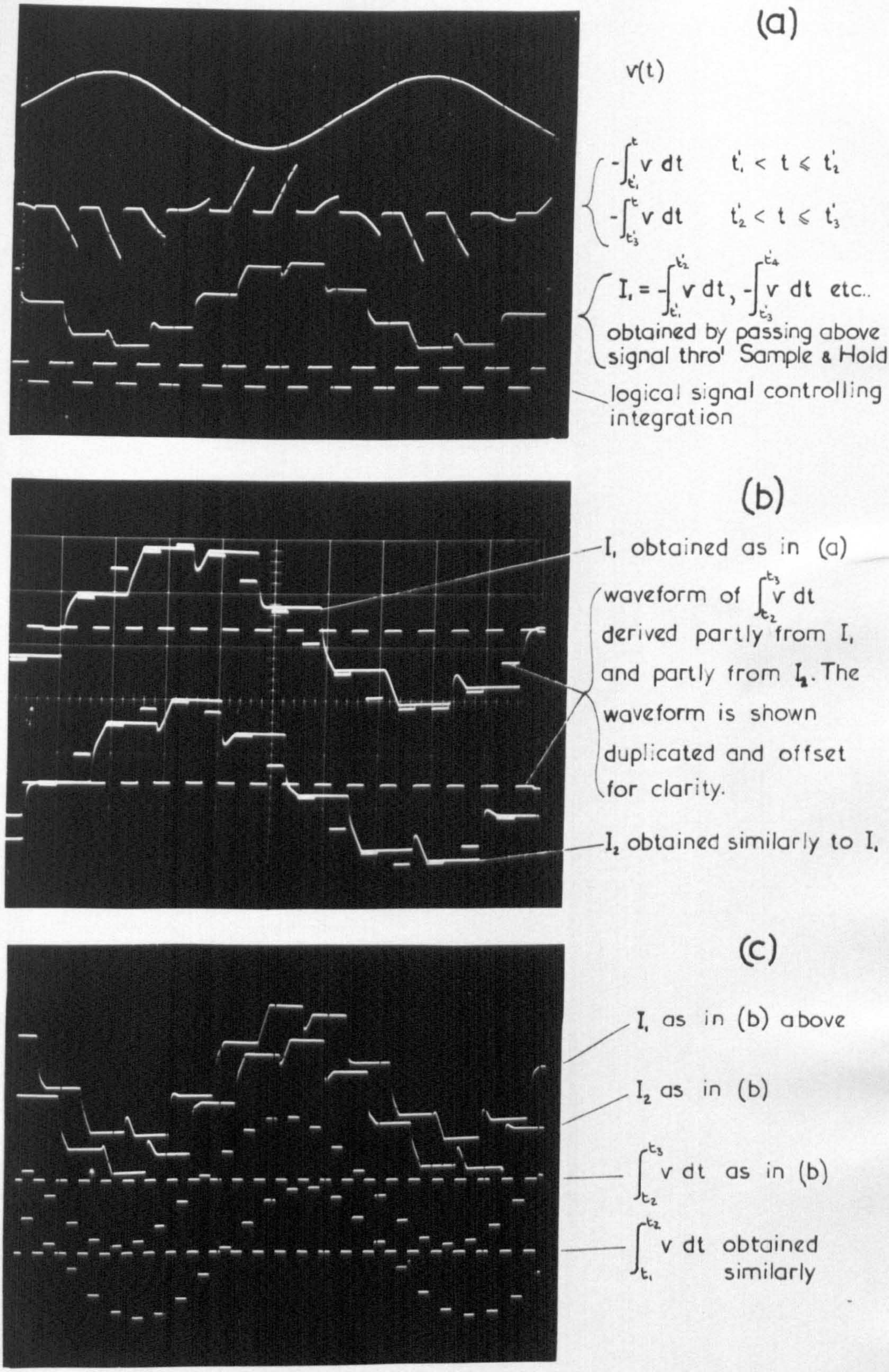


Fig. 5.20 Derivation of Definite Integral Signals

The process starts with the required basic signal (v(t) in this case) passing into two integrator circuits each controlled by the supervisory circuit. The outputs are different and continuously variable with time. The points on these output wave-forms which represent the definite integrals required are located and stored by a Sample and Hold circuit, again suitably supervised. The two signals I_1 and I_2, thus obtained, must be mixed together because each represents part of both the $\int_{t_1}^{t_2} vdt$ and $\int_{t_2}^{t_3} vdt$ signals. This is explained in Fig. 5.20(b). The resultant wave-forms, the end products, are separately displayed in Fig. 5.20(c).

In order to test the relay under transient conditions, and it should be recalled that the equipment is only truly based when the signals are of a transient nature, a test circuit was designed in order that transient wave-forms could be produced regularly and of identical form i.e. the fault occurring at the same point-on-wave. The circuit used is shown in Fig. 5.21. The batch counter was set to trigger at a particular point-on-wave of the input B'N', this voltage being related in phase to the voltage B N in the fault loop mimic. With the batch counter set at 50 the relay P, contained within the batch counter, operates after 50 cycles i.e. 1 second. Automatic resetting after 0.35 second is a feature of the device. The relay operation was arranged to provide first a triggering signal for an oscilloscope and then to make a current path which shorts out a section of the fault loop mimic thus causing transients in both v and i. For development work the circuit was much more convenient than

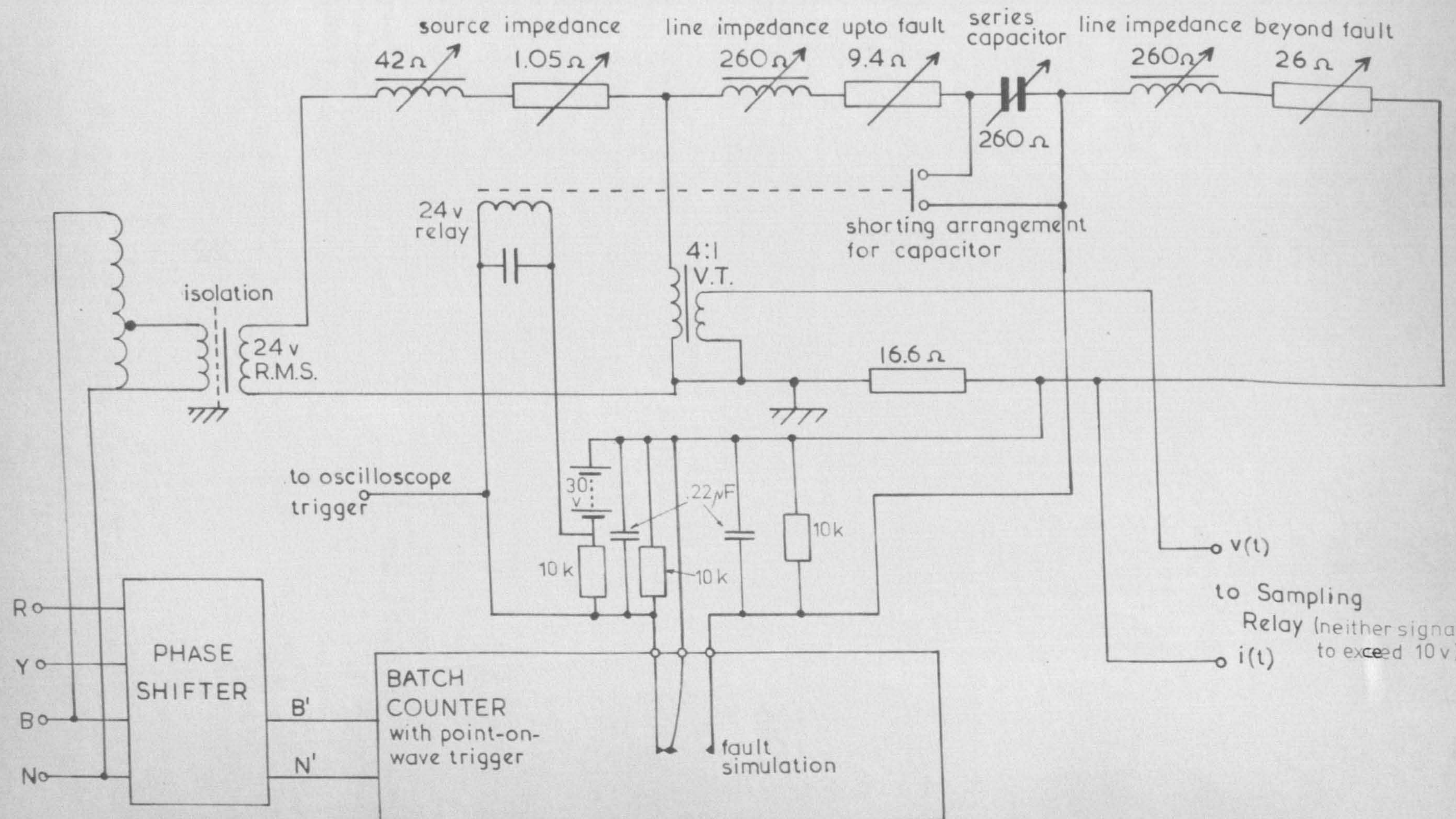


Fig. 5.21 Circuit to Generate i(t) & v(t) for Testing of Sampling Relay

the heavy current test bench which is characterised by a lower fault repetition rate. Oscillograms showing operation of the circuit are shown in Fig. 5.22, the upper and lower sets of signals being identical except that the time scales differ.

The final stage of the sampling relay is the comparator. Since the signals obtained from the arithmetic unit are meaningful for only those times when the Supervisory Control Oscillator is issuing a logical zero ($\bar{u}$ is a logical '1') and is zero at all other times, some smoothing of S_1 and S_2 is desirable before comparison takes place. This smoothing was achieved by passing S_1 and S_2 into Sample and Hold circuits, each supervised by the $\bar{u}$ control signal and then by passing the outputs through L-R high frequency traps. The results of this process were S_1 and S_2 signals, unchanged in phase and magnitude and suitable for use in a simple type of amplitude comparator. The functioning of this special filtering arrangement is shown in Fig. 5.23. In Fig. 5.24, the closing of the relay contacts is shown and corresponds to typical wave-forms of S_1 and S_2. Actual operating speed was variable up to 30 ms. depending upon the composition of the whole fault loop but was typically 20 - 25 ms.

The sampling relay was designed to measure fault position i.e. whether or not the inductance up to the fault is within the relay setting, irrespective of the amount of series capacitance in the fault loop. The characteristics shown in Fig. 5.25 indicate successful measurement of fault position, within 6%

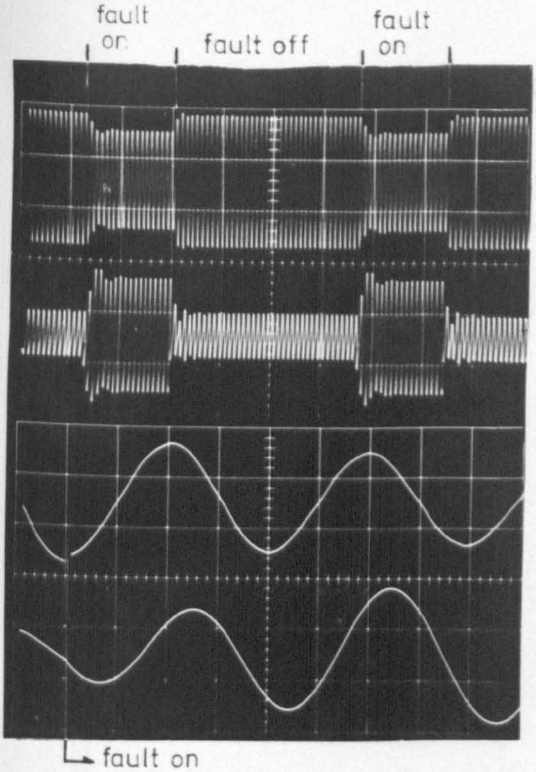


Fig.5.22 Relaying Signals, i & v, Obtained from the Test Circuit of Fig.5.21

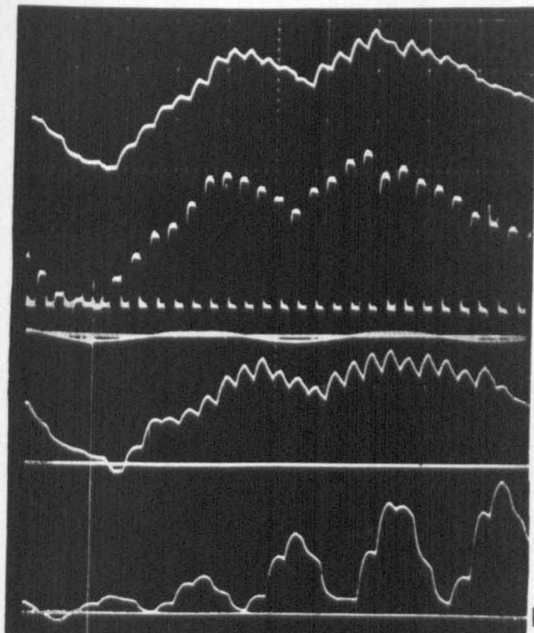


Fig. 5.23 Comparator Signals, S_1 & S_2 Showing Result of Sampling and Smoothing

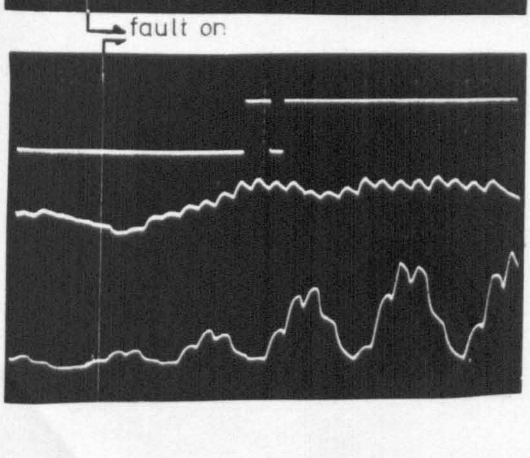


Fig.5.24 Operation of Comparator

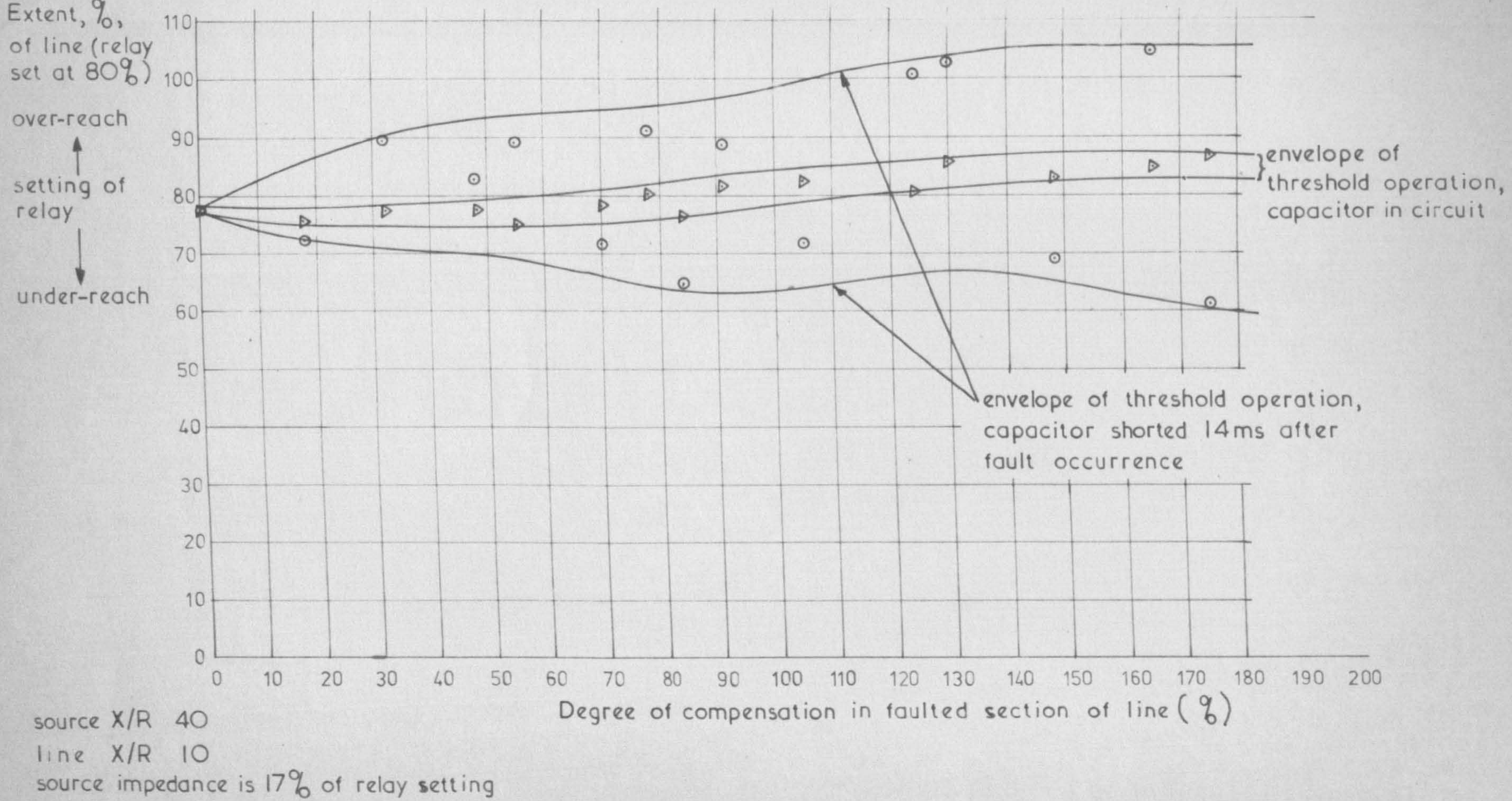


Fig. 5.25 Variation of Relay Accuracy with Amount of Capacitance in Fault Loop

at worst, for a range of capacitance values; the capacitor remaining in circuit throughout the fault. For the case of the capacitor being disconnected during the fault analogous to a protective spark gap operating, the relay also measured fault position but with less accuracy. The corresponding characteristic, also shown in Fig. 5.25, was obtained by arranging an auxiliary relay to close approximately 14 ms. after the fault occurred and thus to short out the series capacitor in the fault loop mimic. This procedure is not exactly equivalent to the action of a protective spark gap which operates when the capacitor voltage is high but the technique is simple and does provide a discontinuity in the fault loop behaviour in a similar way.

6. CONCLUSIONS

A series capacitor can be fairly accurately represented by a simple circuit consisting of two resistors and two capacitors (section 2.2.2.2). This circuit can be used to estimate the amount of energy dissipated in the dielectric under particular circumstances. The energy dissipation in the dielectric of a capacitor which is suddenly discharged is considerably greater than the energy dissipation in a capacitor which is subject to normal loading. The increased thermal stresses contribute to a reduction in capacitor life expectancy and it is concluded that a series capacitor protected by an extinguishing gap is likely to have a shorter life expectancy than a capacitor protected by a simple gap.

Sub-harmonic resonance can arise in power lines containing series capacitors. Because modern power lines have low losses, analyses produced by neglecting resistances are worthwhile. Such treatment indicates clearly how currents containing either harmonic or sub-harmonic components are produced in steady-state when networks containing non-linear inductors are energised from sources providing sinusoidal voltages. The effect of the losses may then be determined using the computer method described in section 3.1.1.2. Transition between normal and sub-harmonic modes are known to occur, and an illustration of how such a change may be produced by closing a switch in a power system has been given (section 3.1.2).

The transient stability analysis developed in section 3.2 was used to show that extinguishing gaps fitted to the series capacitors can be an advantage when spark gap operation occurs for faults external to the compensated line. The type of capacitor over-voltage protection should be selected as follows:

1. Simple gaps should be fitted when either transient stability is not a problem or voltages on the series capacitors caused by all external faults are below the voltage limits tolerable for the capacitor.

2. Extinguishing gaps should be fitted when all the following conditions are true. (a) A transient stability problem exists. (b) The cost of simple gaps and series capacitors of adequate voltage rating is greater than the cost of installing and maintaining extinguishing gaps. (c) The increased cost of capacitors with extinguishing gaps is not so great as to cause another method of maintaining transient stability to be chosen.

When simple gaps operate, the affected capacitors should only be reinstated when the line current has reduced to a value corresponding to a peak capacitor voltage somewhat lower than the spark gap setting. Premature reinstatement may cause spark gaps on previously unaffected capacitors to operate due to mutual coupling between phases.

A digital computer method of achieving the transient analysis of long transmission lines was applied to the problem of analysing lines containing series capacitors and spark gaps (section 3.3.2). Wave-forms were computed for comparison[34] with experimental results and also for comparison with wave-forms calculated assuming zero line susceptance (section 3.3.1.) The characteristics of the results obtained are summarised:

1. Travelling waves introduce significant amounts of high frequency components into sending-end current and voltage wave-forms. When source inductance is present, the voltage wave-form is particularly affected.

2. Operation of a capacitor's extinguishing gap can cause a persistent d.c. offset in the relaying current and also introduces extra travelling waves on to the line.

This continuous presence of both low and high frequency transients in the relaying wave-forms contrasts with the equivalent uncompensated line where all transients are eventually lost due to attenuation. The computed wave-forms were used in the discussion and evaluation of novel distance protection principles (section 4.2).

In series compensated lines, the effect of a transient in the relaying current causes under-reach of impedance measuring distance relays(4.1.3). This is more serious than relay over-reach because faults close to the relay setting may not be recognized. Relay settings should, therefore, be adjusted to take account of this. Impedance measuring distance relays on uncompensated lines tend to over-reach when transients are present in the relaying signals but since these transients are unidirectional in form, filtering in the relays is effective in reducing their effect. The basic transient component of relaying signals in compensated lines is alternating in form,and filtering within the conventional relay may not, therefore be as effective.

A number of different principles have been examined for basing a distance protection scheme (section 4.2). The use of travelling wave-fronts caused by the surges of current towards a fault is not suitable because of the difficulty in distinguishing reflections of wave-fronts from the fault from discontinuities caused by the series capacitor and spark gap (section 4.2.1). The use of a lumped parameter representation of the fault loop offers a number of possibilities. The schemes which require the evaluation of derivatives of current and voltage are not suitable unless the relaying wave-forms are passed through filters capable of removing virtually all noise and distortion. The assumption that resistance is proportional to inductance up to and including the fault, leads to a considerable simplification in the mathematics. The main error caused by this assumption is due to fault impedance. Schemes which made use of integration of current and voltage over definite periods were favoured because of the reduction in the effect of noise and distortion.

A method of measuring fault loop inductance which uses samples of the relaying current and definite integrals of the relaying voltage and currents, was selected for further consideration. For the method to be applied to the problem of measuring inductance in a series R-L-C fault loop, it is necessary for there to be a transient in the relaying wave-forms. It was shown that a transient is always expected (section 4.3.1). As with any scheme whichuses instantaneous values of v and i, it is important that the C.T.s and V.T.'s introduce only small distortions. Because of the errors associated with sampling a distorted wave-form, a practical relay should include initial low-pass filtering of the relaying current or else should incorporate a sampling method which does not select the precise instantaneous values of current. The suitability of a simple sampling circuit was investigated (section 4.3.3.1) by modelling its performance on a digital computer. The digital model could be incorporated into a purely digital sampling relay if desired.

The sampling frequency chosen was a compromise aimed at reducing errors due to travelling wave distortion of v and i wave-forms and obtaining several measurements of fault loop inductance in order that erroneous measurements could be identified. Spurious values should be expected due to the effect of travelling waves, transducer non linearities and operation of the capacitor's protective equipment. The effects of travelling waves and spark gap operation were studied (section 4.3.3.3) by modelling the sampling relay on a digital computer. Identification of erroneous measurements is easily achieved within the digital computer. In analogue equipment, the identification procedure is more complicated; if a means of forming the quotient of S_1 and S_2 is not available then a comparator may be used to compare the signal magnitudes. This should integrate with time the result of the comparison before initiating circuit breaker operation and in this way reduce the effect of erroneous measurements.

The prototype sampling relay which was constructed was rather complex and the quarter square multipliers used were not suitable for handling the wide range of signal amplitudes that are expected in practice. In an application the number of components and therefore the cost and complexity could be reduced and a more suitable multiplying element could be used. The complete scheme for a three phase line (section 5.2) has been given together with operating zones for various fault positions with and without carrier wave assistance. When tested in the laboratory using simulated i and v wave-forms the sampling relay was found to operate in less than 30 ms and it measured with less than 10% error for a wide range of degree of compensation. When the series capacitor was shorted analogous to the spark gap operating, the measuring error was found to be dependent on amount of series capacitance present and the relay over-reached by up to 30% for high levels of compensation.

The use of a small general purpose digital computer instead of the specially constructed analogue sampling relay would cost considerably less in practice, because a single small computer would be able to relay for all faults on a three phase line. The reason for using analogue circuits in the prototype sampling relay is that this type of circuit has been accepted in the protective relay industry. However, the sequencing of circuits and the routing of signals within the relay necessitates a certain amount of digital circuitry for the production of control signals. There are objections against the use of digital circuits in protective relaying because it is thought that they will malfunction in a relay room environment during times of power system faults. This should not be sufficient reason for condemning the analogue version of the sampling relay because the number of digital circuits is so small that special precautions can be employed for potection against failure. As regards the analogue circuits these are inherently reliable because of their negative feedback, but continuous monitoring of their good condition is difficult. The integrity of a digital computer, on the other hand, is relatively easy to monitor continuously.

The digital computer is the most effective equipment for realising the sampling relay because it can easily execute arithmetic division and comparison of numbers, and its operating speed enables it to deal with all relaying functions at a line end without the need for duplication of equipment for different types of fault.

6.1 Future Work

There are two particular aspects of the work described in this thesis which could be extended to good effect.

1. The long line analysis should be used to relate the frequency spectrum found in relaying current and voltage to the fault position, degree of compensation, source impedance, point-on-wave and spark gap setting. It would be useful when considering the performance of new (and existing) power system protective devices, to know what frequencies are present in the basic wave-forms.

2. The use of digital integrated circuits in power system relaying is the big unknown quantity which is holding back relay development in this direction. A number of researchers are working on ways of using the computers[43,44,45], but as far as the author is aware no-one is studying the environment in which the computers must work. Published laboratory work on the immunising of digital circuits to the electromagnetic interference associated with switching stations is much needed.

7. ACKNOWLEDGEMENTS

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APPENDIX 1

List of Principal Symbols.

A	capacitor plate area (section 2.2), a constant elsewhere.
A_j	an alement of A, the angle by which machine j is out of phase with reference machine.
A	column matrix of machine angles.
B	a constant
B	square matrix of conductor geometry (dimension less)
C	capacitance.
C	square matrix of series capacitors.
D	a constant
d	separation of capacitor plates.
E_s	transmission line sending end voltage
E_R	" " receiving " "
E_{jk}	E.M.F. of machine j, phase k.
E_{mj}	peak E.M.F/phase, machine j.
E	column matrix of E.M.F.s.
e	voltage at a point x
e_j	voltage at node j
h	time taken to sample wave-form (section 4.3)
I	primary relaying current (section 4.1.1)
I_s	transmission line sending end current.
I_R	" " receiving " voltage.
I_{jk}	current flowing into machine j, phase k.
I	column matrix of currents.
I_p	" " " phase currents
I_m	" " " modal currents.
i	instantaneous current. In chapters 4 and 5, a signal representing primary relaying current. In section 3321, current at a point x.

i_{jk} current flowing from node j in the direction of node k.

K flux linkage constant (volt.second/(amp)$^{1/3}$).

k relative permittivity (complex)

L linear inductance.

$\mathbf{L}$ square matrix of self inductances

$\mathbf{M}$ square matrix of mutual inductances

m computer program parameter, defined in section 4.3.3.1.

$\mathbf{P}_{elec}$ diagonal matrix of total electrical power input at a source.

$\mathbf{P}_{mech}$ diagonal matrix of total mechanical power input at a source.

p operator d/dt. except in section 4.3.3. where it is number of instantaneous values used to compute sampled value.

$\mathbf{Q}$ column matrix of charge

q charge

R resistance

R_c non-linear resistance representing spark gap (section 3.3)

$\mathbf{R}$ square matrix of resistance.

S_1 sampling relay restrain signal

S_2 " " operate "

s Laplace operator.

t time

Δt increment in t

V primary relaying voltage.

V_c voltage developed across series capacitor.

V_s voltage settings of spark gap

$\mathbf{V}$ column matrix of voltages.

$\mathbf{V}_c$ " " " capacitor voltages.

V_p column matrix of phase voltages.

V_m " " " modal "

v instantaneous voltage. In chapters 4 and 5, a signal representing primary relaying voltage.

X_c reactance of series capacitor at fundamental frequency.

x distance.

Y square matrix of shunt admittances.

Z impedance

Z_c characteristic impedance

Z_N impedance setting of relay.

Z_F impedance between relay point and fault position.

Z_0, Z_1, Z_2 zero, positive, negative sequence impedance.

Z' square impedance matrix of same order as number of wires

Z " " " " " " " " " phase conductors.

α, θ, φ constant angle (radians).

δ capacitor loss angle.

ε permittivity (complex)

ε_o permittivity of free space (real)

μ permiability

v propagation velocity

τ travel time of transmission line (section 3.3.), a time constant elsewhere.

ω angular frequency.

APPENDIX 2.1

Calculation of Effect of Earth-Return Path

The contribution to the line impedance matrix due to earth-return path is $R_e + jX_e$. This is calculated using infinite series developed by Carson.[17] The calculation is best arranged for computation as follows.

Firstly are defined:

$r_{ij} = \left[\frac{\omega.\mu}{\rho}\right]^{\frac{1}{2}} . D_{ij}$, θ_{ij} is the angle subtended at the i th conductor by the images of the i th and j th conductors in the earth plane. D_{ij} is the distance between the i th conductor and the j th image.

now, $R_e = \frac{\omega\mu}{\pi} . P$ and $X_e = \frac{\omega\mu}{\pi} . Q$

for $r \leqslant 5$,

$$P_{ij} = \frac{\pi}{8}(1-S_4) + \tfrac{1}{2}\log\left(\frac{2}{\gamma r}\right)S_2 + \tfrac{1}{2}\theta S_2' - \frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{2} + \frac{\sigma_3}{\sqrt{2}}$$

$$Q_{ij} = \tfrac{1}{4} + \frac{\log\left(\frac{2}{\gamma r}\right)(1-S_4)}{2} - \frac{\theta S_4'}{2} + \frac{\sigma_1}{\sqrt{2}} - \frac{\pi S_2}{8} + \frac{\sigma_3}{\sqrt{2}} - \frac{\sigma_4}{2}$$

$\gamma = 1.7811$ (Euler's constant)

$$S_2 = \sum_0^{\infty} a_n \cos(4_n + 2)\theta$$

$$S_2' = \sum_0^{\infty} a_n \sin(4_n + 2)\theta$$

$$S_4 = \sum_0^{\infty} c_n \cos(4_n + 4)\theta$$

$$S_4' = \sum_0^{\infty} c_n \sin(4_n + 4)\theta$$

$$\sigma_1 = \sum_0^{\infty} e_n \cos(4n + 1)\theta \,, \quad \sigma_2 = \sum_0^{\infty} g_n (S_2)_n$$

$$\sigma_3 = \sum_{0}^{\infty} f_n \cos(4n+3)\theta , \quad \sigma_4 = \sum_{0}^{\infty} h_n (S_4)_n$$

and $$a_n = \frac{-a_{n-1}}{2n(2n+1)^2(2n+2)} \left(\frac{r}{2}\right)^4, \quad a_o = \frac{r^2}{8}$$

$$c_n = \frac{-c_{n-1}}{(2n+1)(2n+2)^2(2n+3)} \left(\frac{r}{2}\right)^4, \quad c_o = \frac{r^4}{192}$$

$$e_n = \frac{-e_{n-1}}{(4n-1)(4n+1)^2(4n+3)} r^4, \quad e_o = \frac{r}{3}$$

$$f_n = \frac{-f_{n-1}}{(4n+1)(4n+3)^2(4n+5)} r^4, \quad f_o = \frac{r^3}{45}$$

$$g_n = g_{n-1} + \frac{1}{4n} + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{4n+4}, \quad g_o = \frac{5}{4}$$

$$h_n = h_{n-1} + \frac{1}{4n+2} + \frac{1}{2n+2} + \frac{1}{2n+3} - \frac{1}{4n+6}, \quad h_o = \frac{5}{3}$$

for $r > 5$,

$$P_{ij} = \frac{\cos\theta}{\sqrt{2}\, r} - \frac{\cos 2\theta}{r^2} + \frac{\cos 3\theta}{\sqrt{2}\, r^3} + \frac{3\cos 5\theta}{\sqrt{2}\, r^5}$$

$$Q_{ij} = \frac{\cos\theta}{\sqrt{2}\, r} - \frac{\cos 3\theta}{\sqrt{2}\, r^3} + \frac{3\cos 5\theta}{\sqrt{2}\, r^5}$$

APPENDIX 2.2

Analysis of Capacitor Discharge

The circuit of Fig. 2.10(a) is analysed.

$$\frac{q_1}{C_1} + R\frac{dq_1}{dt} + R\frac{dq_2}{dt} = 0$$

$$R\frac{dq_1}{dt} + \frac{q_2}{C_2} + (R+R_2)\frac{dq_2}{dt} = 0$$

taking Laplace Transforms gives:

$$\begin{bmatrix} \frac{1}{C_1} + sR & sR \\ sR & \frac{1}{C_2}+s(R+R_2) \end{bmatrix} \times \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} R(q(0)+q_2(0)) \\ R(q(0)+q_2(0))+R_2q_2(0) \end{bmatrix}$$

solving for Q_2 gives:

$$Q_2 = \frac{sRR_2q_2(0) + \frac{1}{C_1}(R(q(0) + q_2(0))+R_2q_2(0)}{s^2RR_2 + s\frac{RC_1 + RC_2 + R_2C_2}{C_1C_2} + \frac{1}{C_1C_2}}$$

In the particular case of interest in section 2.2.2.2.6,

$R = 0.05\,\Omega$, $R_2 = 0.52\Omega$, $C_1 = 0.56\ \mu F$, $C_2 = 0.44\ \mu F$.

If initially both C_1 and C_2 are charged to 1 volt.

then $Q_2 = \left[\frac{1.011}{(s + 4.10^6)} - \frac{0.011}{(s + 40.10^6)}\right].q_2(0)$

inverse transforming gives:-

$$q_2(t) = (1.011\ e^{-4t} - 0.011\ e^{-40t}).q_2(0);\text{t is in } \mu s.$$

then $i_2(t) = (0.440\ e^{-40t} - 4.044\ e^{-4t}).q_2(0).10^6$ amps.

APPENDIX 3.1

Derivation of the Two-Port Equations for a Transmission Line with Line Losses Taken into Account

The circuit of Fig. A3.1 is considered. This circuit is exactly equivalent to Fig. 3.11f except that the system of subscripts has been changed slightly

for the circuit of Fig. A1

$$e_1 = e'_1 - \frac{R}{4}.i_{12}.$$

$$e_2 = e'_2 - \frac{R}{4}.i_{21}.$$

$$e_3 = e'_3 - \frac{R}{4}.i_{34}$$

$$e_4 = e'_4 - \frac{R}{4}.i_{43}$$

Remembering that the travel times for each line in Fig. A3.1 is $\tau/_2$ the above equations can be substituted into equations 321 to obtain directly.

$$i_{12}(t).a = y\,e'_1(t) + I_1(t-\tau/2) \quad \ldots\ \ldots 1.$$

$$i_{21}(t).a = y\,e'_2(t) + I_2(t-\tau/2) \quad \ldots\ \ldots 2.$$

$$I_1(t-\tau/2) = -y\,e'_2(t-\tau/2) - i_{21}(t-\tau/2).b \quad \ldots 3.$$

$$I_2(t-\tau/2) = -y\,e'_1(t-\tau/2) - i_{12}(t-\tau/2).b \quad \ldots 4.$$

where $a = 1 + \frac{R}{4}\cdot y$

$b = 1 - \frac{R}{4}\cdot y, \; y = \sqrt{\frac{C_1}{L_1}}$

by symmetry

$$-\,i_{21}(t).a = y\,e'_2(t) + I_3(t-\tau/2) \quad \ldots\ \ldots 5.$$

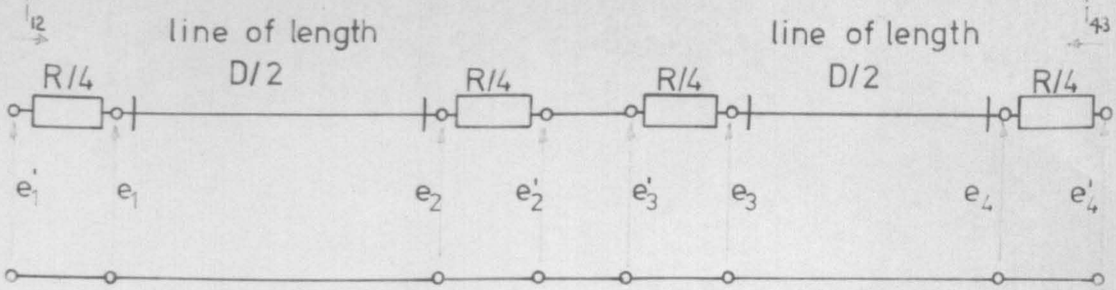

Fig. A3.1 Circuit Used in Derivation of Two-port Equations of Lossy Transmission Line

$$i_{43}(t).a = ye'_4(t) + I_4(t - \tau/2) \quad .. \quad .. \quad .. \quad 6.$$

$$I_3(t-\tau/2) = -ye'_4(t-\tau/2)-i_{43}(t-\tau/2).\ b \quad .. \quad 7.$$

$$I_4(t-\tau/2) = -ye'_2(t-\tau/2)+i_{21}(t-\tau/2).\ b \quad .. \quad 8.$$

substituting $t = t-\tau/2$ into equations 1-8 we obtain.

$$i_{12}\ (t-\tau/2)a = ye'_1(t-\tau/2) + I_1(t-\tau) \quad .. \quad 9.$$

$$i_{21}\ (t-\tau/2)a = ye'_2(t-\tau/2) + I_2(t-\tau) \quad .. \quad 10.$$

$$I_1\ (t-\tau) = -ye'_2(t-\tau) - i_{21}(t-\tau).b \quad .. \quad 11.$$

$$I_2\ (t-\tau) = -ye'_1(t-\tau) - i_{12}(t-\tau).b \quad .. \quad 12.$$

$$-\ i_2\ (t-\tau/2)a = ye'_2\ (t-\tau/2)+ I_3(t-\tau) \quad .. \quad 13.$$

$$i_{43}\ (t-\tau/2)a = ye'_4\ (t-\tau/2)+ I_4(t-\tau) \quad .. \quad 14.$$

$$I_3\ (t-\tau) = -ye'_4(t-\tau) - i_{43}(t-\tau).b \quad .. \quad 15.$$

$$I_4\ (t-\tau) = -ye'_2(t-\tau) + i_{21}(t-\tau).b \quad .. \quad 16.$$

10 - 13 gives $2\ i_{21}(t-\tau/2).a = I_2(t-\tau) - I_3(t-\tau)$

10 + 13 gives $2y\ e'_2(t-\tau/2) = I_2(t-\tau) - I_3(t-\tau)$

substitute into 3: $I_1(t-\tau/2) = (I_2(t-\tau).(1 -\frac{b}{a})+I_3(t-\tau)(1+\frac{b}{a}))/2$

substitute this into 1: $i_{12}(t) = y/a.e'_1(t) + I'_1(t-\tau)$

where $I'_1(t-\tau) = \frac{1}{2a}\ ((1+h)I_3(t-\tau)+(1-h)I_2(t-\tau))$

& $\quad I_3(t-\tau) = -ye'_4(t-\tau) - i_{43}(t-\tau).b$

& $\quad I_2(t-\tau) = -ye'_1(t-\tau) - i_{12}(t-\tau).b$ where $h = \frac{b}{a}$

substituting $e_1 = e'_1$, $e_2 = e'_4$, $i_{21} = i_{43}$ and the equations are in final form:-

$$i_{12}(t) = \frac{1}{Z}e_1(t) + I_1(t-\tau)$$

$$I_1(t-\tau) = \left(\frac{1+h}{2}\right)\left(-\frac{1}{Z}e_2(t-\tau) - i_{21}(t-\tau).h\right)$$

$$+ \left(\frac{1-h}{2}\right)\left(-\frac{1}{Z}e_1(t-\tau) - i_{12}(t-\tau).h\right)$$

$$\text{where } Z = \sqrt{\frac{L_1}{C_1}} + R/4$$

the equations for $i_{21}(t)$ and $I_2(t-\tau)$ are obtained directly by recognising symmetry.

APPENDIX 4.1

Derivation of Formula for Calculating Filtered Sampled Value of A at time t

The function A is approximated by a series of step functions as shown in Fig. 4.11. A derived function X_j is obtained analogous to the signal, A, being imposed on an R-C circuit at time (t-h-h/p). The voltage X_j, present on the capacitor after the first period of h/p would then be y x A(t-h) i.e. the fraction y = 1/m of the difference between the impressed step function and the original capacitor voltage, hence:-

$$X_p = y.\ A(t-h)$$

$$X_{p-1} = X_p + y\left(A\left(t-\frac{h(p-1)}{p}\right) - X_p\right)$$

$$= X_p.(1-y) + y.\ A\left(t-\frac{h(p-1)}{p}\right)$$

$$= y\left(A(t-h).(1-y) + A\left(t-\frac{h(p-1)}{p}\right)\right)$$

$$X_{p-2} = y\left(A(t-h).(1-y)^2 + A\left(t-\frac{h(p-1)}{p}\right).(1-y) + A\left(t-\frac{h(p-2)}{p}\right)\right)$$

in general

$$X_{p-n} = y.\sum_{j=p-n}^{j=p} A\left(t-\frac{jh}{p}\right).(1-y)^j$$

with time t corresponding to n = p

filtered sampled value of A at time t

$$= X_o = \sum_{j=o}^{j=p} A\left(t-\frac{jh}{p}\right).(1-y)^j.\ y$$

$$= \sum_{j=o}^{j=p} A\left(t-\frac{jh}{p}\right).\ \frac{\left(1-\frac{1}{m}\right)^j}{m}$$

after substituting $y = \frac{1}{m}$.

$$R_2 i_2^2(t) = (0.194\, e^{-80t} + 16.0 e^{-8t} - 1.76 e^{-44t}) . q_2^2(0) . 10^{12} . 0.52$$

energy lost in R_2(the dielectric) $= \int_0^\infty R_2\, i_2^2\, dt$

$$= \left(\frac{0.194}{80} + \frac{16}{8} - \frac{1.76}{44}\right) q_2^2(0) . 10^6 .\ 0.52$$

$= 0.202\,\mu$joules.

total energy in capacitor before discharge $= \frac{1}{2}.C.V^2 = 0.5\,\mu$joules

Therefore, proportion of energy dissipated in dielectric is 40.4%.